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Hybrid optimization of EDLP and high-low pricing strategies

Hamed Karimi[a*]

[a]*School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran*

C H R O N I C L E

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A B S T R A C T

In today's fiercely competitive retail landscape, implementing effective pricing strategies is critical not only for boosting sales but also for securing a larger market share and ensuring long-term business sustainability. The ability to capture a greater share of the market directly influences a retailer's positioning and competitive edge, making pricing decisions pivotal. This paper introduces a hybrid optimization model that strategically combines Everyday Low Pricing (EDLP) and High-Low Pricing (HL) strategies, designed to address the intricacies of dynamic retail markets. The model is initially formulated as a nonlinear optimization problem aimed at maximizing sales to increase market share, all while maintaining profitability within a predefined threshold to ensure the retailer does not incur losses. To enhance the model's practical applicability, particularly in small-scale scenarios, the nonlinear problem is transformed into a Mixed-Integer Programming (MIP) model, facilitating its solvability. However, as retail applications scale up, the computational complexity becomes more challenging, necessitating the use of the Grey Wolf Optimization (GWO) algorithm. The GWO algorithm effectively balances computational efficiency with solution quality, making it a robust approach for large-scale problems. A significant contribution of this research is the linearization of the model under conditions where the products designated for High-Low pricing (referred to as 'Golden' products) are predetermined by the retailer. This linearization simplifies the computational process, enabling the model to scale and be applied in large retail settings. Developed in collaboration with a major Iranian supermarket chain, the model leverages real-world data to optimize discount levels and timing across various product categories. Extensive numerical experiments demonstrate the model's effectiveness in increasing sales, thereby contributing to a larger market share while ensuring that profitability remains within acceptable bounds. By providing actionable insights and strategic recommendations, this research offers a practical, scalable solution for optimizing retail pricing strategies in a data-driven and competitive environment, ultimately supporting retailers in their quest to dominate the market.

1. Introduction

In today's competitive world, optimizing discounts has become one of the most crucial strategic tools for retailers to increase sales and improve profitability. Discounts are not only an effective way to attract new customers but can also encourage existing customers to make more frequent and larger purchases. However, determining the optimal discount level and timing poses a significant challenge for store managers due to the complex effects they have on demand, product inventory, and even customer experience (Guchhait et al., 2024). Moreover, adhering to various business rules and constraints, which may be set by the board or suppliers, plays a crucial role in the decision-making process. Therefore, the use of data-driven models and optimization algorithms is of great importance in making optimal decisions in this area (Chen, Mersereau, & Wang, 2012).

One of the main challenges for retailers in this context is choosing the appropriate pricing strategy for their products. Two major strategies in this area are Everyday Low Pricing (EDLP) and High-Low Pricing (HL). In the EDLP strategy, retailers

* Corresponding author.
E-mail address: hamedkarimi7676@ut.ac.ir (M. Karimi)

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offer stable and relatively low prices for their products that experience little change over time. This approach helps build customer trust, as customers are confident that they can always purchase the products they need at a reasonable price without worrying about price fluctuations (Pechtl, 2004). On the other hand, in the HL strategy, prices change regularly, and deep discounts are offered during specific periods. This method stimulates demand during discount periods and attracts customers to make immediate and larger purchases, but it can also lead to severe demand fluctuations and decreased customer trust (Ellickson & Misra, 2008).

The choice between these two pricing strategies has a significant impact on retailers' performance. For example, some stores have experienced increased sales by switching from the EDLP strategy to HL, while a shift from HL to EDLP may lead to decreased profitability. For instance, a study showed that switching from HL to EDLP could reduce retailer profits, although it might also increase sales volume for suppliers (Hoch, Drèze, & Purk, 1994). The experience of the JC Penney retail chain is a prominent example that illustrates how a shift from HL to EDLP might yield unfavorable results. This change not only failed to improve the store's performance but also led to a sharp decline in the company's stock value and financial performance (Bailey, 2008).

Given these challenges, this paper aims to develop an optimization model capable of combining both EDLP and HL strategies in a hybrid environment. This model, directly inspired by a collaboration with a large Iranian supermarket chain, is designed to increase sales and capture a larger share of the retail market at a specified profitability level. It seeks to provide optimal decisions on discount levels and timing by leveraging real data. The model considers the interactions between products, inventory constraints, and various business rules, helping retailers perform better in a dynamic and competitive market. This research builds on previous studies in pricing and discount optimization and strives to offer valuable practical and theoretical insights by developing a hybrid model. The proposed model analyzes existing data and offers optimal strategies to attract customers and increase market share. The findings of this research are expected to help retailers optimize their pricing and discount processes using innovative and data-driven approaches, thereby achieving greater success in competitive and volatile market environments.

The remainder of this paper is organized as follows. Section 2 provides a comprehensive review of the related literature, discussing key studies that have explored various pricing strategies, including Everyday Low Pricing (EDLP) and High-Low (HL) pricing. In Section 3, the problem is defined, and the challenges associated with integrating EDLP and HL strategies in a dynamic retail environment are outlined. Section 4 presents the mathematical formulation of the proposed hybrid optimization model, focusing on the initial nonlinear formulation. Section 5 introduces the solution methods, including the transformation of the nonlinear model into a Mixed-Integer Programming (MIP) model and the application of the Grey Wolf Optimization (GWO) algorithm for solving large-scale instances. Section 6 presents computational examples and results, demonstrating the effectiveness of the proposed model in real-world retail scenarios. Finally, Section 7 summarizes the key conclusions of the study and suggests directions for future research, highlighting areas where further investigation could enhance the understanding and application of hybrid pricing strategies.

2. Literature review

Pricing strategies in retail, particularly the interplay between High-Low (HL) pricing and Every-Day Low Pricing (EDLP), have been extensively studied due to their significant impact on demand, profitability, and market competition. Understanding the nuances of these strategies and their implications under various market conditions is crucial for retailers aiming to optimize sales and maintain profitability. This literature review explores key studies that have contributed to the understanding of these strategies, highlighting the research gap that this paper addresses.

Breiter and Huchzermeier (2015) provide a foundational analysis of the challenges associated with demand forecast errors in HL pricing strategies. They propose a two-segment demand forecasting approach combined with supply contracts as a solution. Their study emphasizes that while demand is sensitive to past retail prices, forecast errors remain inevitable due to unpredictable competitive promotions. This research underscores the complexity of HL pricing, particularly in terms of forecasting demand accurately amidst fluctuating competitive actions. In a more recent study, Rekha et al. (2024) expand on the optimization of joint profit within the context of sustainable development goals. Their work involves a solution procedure that determines the optimal decisions under risk, demonstrating the need to integrate sustainability into pricing strategies. This approach is particularly relevant when considering how environmental and social factors can influence both demand and pricing strategy effectiveness.

Qureshi and Lazam (2024) address the limitations of current dynamic pricing models, specifically in the context of ride-hailing applications. They critique reinforcement learning-based approaches and introduce a hybrid model utilizing a classification and regression tree (CART) algorithm. While their focus is on the ride-hailing industry, the study's insights into the shortcomings of traditional dynamic pricing models are applicable to broader retail contexts, particularly in optimizing prices based on external factors such as competitor actions and market conditions. Further exploration of pricing strategies is provided by Meng Wu et al. (2022), who investigate the influence of strategic consumer behavior on different pricing approaches. Their comparative analysis of fixed pricing, strategic high pricing, and HL pricing reveals that HL pricing is only beneficial when markdown discounts are relatively small. Conversely, in markets with a low proportion of strategic consumers or where significant markdowns are required, fixed pricing may be more profitable. This study highlights the importance of understanding consumer behavior in determining the optimal pricing strategy, suggesting that the effectiveness of HL pricing is contingent on the market's specific characteristics.

He et al. (2024) explore the response of retail prices in supermarkets to fluctuations in commodity prices during 2007-2009, comparing EDLP stores with those that frequently offer sales. Their findings indicate that EDLP stores, which avoid frequent price changes, are less likely to adjust prices in response to commodity price shifts. In contrast, other stores exhibit symmetric but infrequent price adjustments. This study adds to the understanding of how external economic factors, such as commodity prices, influence the stability and predictability of pricing strategies, particularly in an EDLP context. Hamilton (2024) provides a comprehensive review of consumer price evaluation strategies, including internal reference prices (IRPs), external reference prices (ERPs), and price images (PIs). His work emphasizes the psychological mechanisms behind price perception and the factors that determine which pricing strategy consumers use. This is crucial in understanding why certain pricing strategies, such as HL or EDLP, may resonate differently with various consumer segments, thereby influencing the overall success of these strategies in practice.

Hamdani (2022) and Nouri-Harzvili and Hosseini-Motlagh (2023) further delve into the impact of discounts on brand reputation and inventory management. Hamdani's research shows that higher discounts can enhance brand reputation and build customer loyalty, while Nouri-Harzvili and Hosseini-Motlagh focus on the dynamics of pricing in online retail, emphasizing the need to balance discount levels with inventory management. These studies illustrate the multifaceted nature of pricing strategies, where not only sales but also brand equity and operational efficiency must be considered.

In the context of customer satisfaction, Ilyas et al. (2022) examine how pricing, advertising, and service quality influence customer satisfaction within support services. Their findings confirm that pricing strategies are directly related to customer satisfaction, a factor that is critical for long-term success in competitive markets. Meanwhile, Mohammadi-Pour et al. (2023) develop a model for optimizing sales promotions in retail markets, showing that ignoring competition in promotion planning can lead to significant profit losses. Their work, which utilizes non-linear integer programming, highlights the importance of competitive dynamics in shaping effective promotional strategies.

Khanlarzade and Farughi (2023) explore the impact of hybrid pricing strategies within a deteriorating supply chain, using a Stackelberg game framework. They introduce a novel algorithm based on Bayesian conjugate pairs to optimize production rates under bounded rationality. Their findings underscore the importance of transparency and information availability in hybrid pricing strategies, particularly in supply chains where leader-follower dynamics are critical.

Cohen-Hillel, Panchamgam, and Perakis (2022) contribute to the discussion by exploring dynamic promotion planning using Bounded-Memory Peak-End demand models. They demonstrate that under specific conditions, a HL pricing strategy can be optimal, particularly when current prices dominate past and competitor prices in influencing demand. Their approach, which combines dynamic programming with a Polynomial-Time Approximation Scheme (PTAS), shows a significant increase in revenue, thus validating the effectiveness of HL strategies in certain retail scenarios.

Shah (2017) provides a cautionary tale of JC Penney's failed shift from an HL pricing strategy to EDLP. The study attributes this failure to the company's inability to align its marketing mix with the new strategy, leading to poor consumer acceptance. This highlights the importance of understanding consumer expectations and the risks involved in shifting between pricing strategies without adequate market research and adjustment. In summary, while there is substantial research on the effectiveness of both HL and EDLP strategies, a clear research gap exists in understanding how a combination of these strategies can be optimized in dynamic retail environments. This study aims to fill this gap by proposing a model that integrates both strategies, taking into account factors such as consumer behavior, market conditions, and the competitive landscape, with the ultimate goal of maximizing sales while maintaining profitability

3. Problem Description

In recent years, promotion optimization has emerged as a critical focus in sales and marketing management, especially in competitive markets and price-sensitive environments where customers react strongly to price changes. Two predominant strategies in this area are High-Low Pricing and Everyday Low Pricing (EDLP). Each of these strategies comes with its own set of advantages and disadvantages, and the choice between them can significantly influence a company's revenue and market share.

High-Low Pricing is a dynamic approach where products are periodically offered at substantial discounts, encouraging customers to purchase during specific promotional periods. One advantage of this strategy is its ability to attract a larger customer base during these periods, creating a sense of urgency to buy. This method enables stores to clear excess inventory and maximize sales volumes within targeted timeframes. However, this approach also has its downsides, such as demand instability and significant fluctuations in cash flow. Additionally, customers might delay their purchases at regular prices, waiting for the next discount, which can negatively impact overall profitability (Jobber et al., 2012).

In contrast, Everyday Low Pricing (EDLP) is a strategy that maintains stable and consistently low prices over time, aiming to attract price-sensitive customers. This approach builds customer trust, as they know they do not need to wait for periodic discounts and can always buy at a fair price. The benefits of EDLP include reduced advertising and marketing costs due to the absence of frequent promotional campaigns, as well as increased customer loyalty. However, this strategy may lack the excitement and appeal of discount-driven shopping, which can potentially lead to lower long-term profitability, as it often operates on thinner margins and may struggle to capitalize on sudden spikes in demand (Yang et al., 2015).

Given the challenges associated with both strategies, this research explores the potential of combining them into a hybrid model. The primary goal is to develop an optimization model that integrates the strengths of both High-Low and EDLP strategies, while mitigating their respective weaknesses. A key aspect of this model is its consideration of inventory effects. Many customers respond to periodic discounts by purchasing and stockpiling goods in larger quantities, which can significantly impact demand in subsequent periods (Cohen et al., 2017). Building on previous studies and optimization models in promotions and pricing—such as those by Cohen, Kalas, and Perakis (2021) and Joshi & Bhatt (2021)—this research delves into the combined effects of High-Low and EDLP strategies on profitability and customer satisfaction. The anticipation is that this hybrid strategy will lead to an optimal solution capable of effectively operating in dynamic market environments.

In this study, we propose an optimization model that combines High-Low and EDLP strategies to maximize sales and market share for a retail store. The model assumes that the store offers K products from C product categories and seeks to determine the optimal discount levels for these products over t time periods. The proposed model determines which products should be priced using the EDLP strategy and which should utilize the High-Low strategy in each period. This optimization enables the store to make the best decisions for each product in each period based on prevailing market conditions and customer demand. In this model, each product can be selected as a "Golden" item only once during the t time periods. This restriction is imposed for several reasons. Firstly, if a product is repeatedly selected as a Golden item, the value and effectiveness of its discounts may diminish over time. Customers may become accustomed to frequent discounts, reducing the product's appeal and potentially lowering its profit margin. Secondly, this limitation ensures that discount opportunities are distributed more evenly across different products, allowing all items to benefit from attractive discounts. Furthermore, in each time period, a maximum of GCN products from each product category C can be selected as Golden items. This limitation also serves a logical purpose: if too many products from the same category are discounted simultaneously, it could lead to market saturation, diminishing the effectiveness of discounts within that category. By imposing this restriction, the model strategically and intelligently distributes discounts across various product categories, preventing excessive competition within a single category and maintaining the overall appeal of the store.

The proposed model aims to optimize the combination of these two strategies to achieve the set objectives. The primary objective function in this model is to maximize sales, a critical factor in increasing the store's market share. In competitive markets, gaining a larger market share is crucial for long-term growth and sustainability. Increased sales not only boost market share but also provide opportunities to build long-term relationships with new customers and strengthen connections with existing ones. However, maintaining profitability is equally important and should not be sacrificed for increased sales. Therefore, profitability is incorporated as a constraint within the model. This constraint ensures that the store's profit does not fall below a certain threshold, thereby maintaining a balance between growing sales and preserving financial stability. This approach allows the store to strive for market share growth while also ensuring its financial health. The model incorporates a business rule that ensures the overall average discount offered by the store does not exceed a predetermined threshold. This rule is typically set by the company's board of directors to control costs and maintain profit margins. In chain retail stores, it is common practice to establish a maximum limit for the average discount across all products, ensuring that the store's overall discounting strategy remains sustainable and does not undermine profitability. This constraint helps the store balance attracting customers through discounts while safeguarding against unnecessary revenue and profit losses.

4. Mathematical formulation

Sets & Indices:

T	Periods	$t \in \{1, 2, \dots, T\}$ (weeks)
O	Ordinary products	$k \in \{1, 2, \dots, O\}$ (SKUs)
G	Golden products	$k' \in \{1, 2, \dots, G\}$ (SKUs)
K	Total products $K = O \cup G$	$k \in \{1, 2, \dots, K\}$ (SKUs)
Q	Cross-item products	$q \in \{1, 2, \dots, q\}$
C	Product groups	$c \in \{1, 2, \dots, c\}$
L	The promotion discount options	$l \in \{1, 2, \dots, l\}$ (percentage)

Parameters:

b_{kt}	The base demand of product $k \in K$ in period $t \in T$ when there is no promotion
c_{kt}	The consumer price of the product $k \in K$ in period $t \in T$
p_{kt}	The profit margin of the product $k \in K$ in period $t \in T$

α^l_{kt} The promotion discount amount of option l for product $k \in K$ in period $t \in T$

λ^l_{kt} The coefficient of increase in sales volume due to the discount option l for product $k \in K$ in period $t \in T$

β^{lq}_{kt} The percentage of inter-product effects of product q due to the discount of product $k \in K$

θ_k Effect of saving SKU k in further periods $k \in K$

TP_t The Total expected profit in period T

w_{kt} The discount commitment of suppliers to product $k \in K$ in period $t \in T$

U_{kt} The maximum amount allowed promotion for product $k \in O$ in period $t \in T$

GN_t The amount Of Golden Products in period $t \in T$

GNC_{tc} The amount Of Golden Products From Product group $c \in C$ in period $t \in T$

$U'_{k\prime t}$ The maximum amount allowed promotion for golden product $k \in G$ in period $t \in T$

L_{kt} The minimum amount allowed promotion for product $k \in O$ in period $t \in T$

$L'_{k\prime t}$ The minimum amount allowed promotion for golden product $k \in G$ in period $t \in T$

a The maximum average discount allowed for all products in T periods

Variables:

D_{kt} The effected demand of product $k \in K$ in period $t \in T$ when there is a promotion

Pr_{kt} The promotion discount of the product $k \in K$ in period $t \in T$

x^l_{kt} A binary variable equal to 1 if the promotion discount α_l is selected for the product $k \in K$ in period $t \in T$

y^c_{kt} A binary variable equal to 1 if product $k' \in G$ of category $c \in C$ is selected for the golden discount at time $t \in T$

Part I: Profit from each product

If there is no discount, $c_{tk}p_{tk}$ is the profit per product unit for the store. Now, if a discount is considered for product k, its profit will be reduced by the amount of discount considered.

Therefore, the profit of the product per unit is equal to c_{tk} $(p_{tk} - pr_{tk})$. Since the amount of product discount is a variable to be optimized, the mentioned relationship is displayed as follows:

$$c_{kt}(p_{kt} - \sum_l \alpha^l_{kt}x^l_{kt} + w_{kt}) \quad \forall\, k \in K, t \in T, l \in L \tag{1}$$

Part II: Effective demand of each product

Three effects are evident in the demand for discounted products and their substitutes/complement. In the inter-product effect, the discounted product takes part of the demand for substitutes or increases the sales of complementary products. In addition, discounts can encourage consumers to save by purchasing larger quantities than usual. Also, past promotions can reduce the demand for the product (stockpiling effect). The following relationship can be used to create the effective demand function:

$$D_{kt} = b_{kt} + \sum_l \lambda^l_{kt}b_{kt}\alpha^l_{kt}x^l_{kt} + \sum_l \sum_q \beta^{lq}_{kt}b_{kt}\alpha^l_{kt}x^l_{qt} + \sum_{g,l} \alpha^l_{kt}x^l_{kt}\, pr_{kt}\theta_k \left(\frac{1}{2}\right)^{t-1} \forall\, k \in K, t \in T, l \in L \tag{2}$$

The first term of the above relationship is the basic demand. The second term shows the effect of changing demand based on past promotions. In the third term, the effect of substitution or complementarity of goods is considered.

Therefore, we need to solve the discount optimization problem as follows:

$$max\ Z = D_{kt} \quad \forall\, k \in K, t \in T \tag{3}$$

$$\sum_l x^l_{kt} = 1 \quad \forall\, k \in K, t \in T \tag{4}$$

$$\sum_{t \in T} y^c_{kt} = 1 \quad \forall k' \in G, c \in C \tag{5}$$

$$\sum_{c \in C} \sum_{k\prime \in G} y^c_{kt} = GN_t \quad \forall t \in T \tag{6}$$

$$\sum_{k\prime\in G} y_{kt}^{c} = GNC_{tc} \qquad \forall t \in T, c \in C \qquad (7)$$

$$Pr_{kt} = \sum_{l} \alpha_{kt}^{l} x_{kt}^{l} \qquad \forall k \in K, \forall t \in T \qquad (8)$$

$$L'_{k\prime} - M(1 - y_{k\prime t}^{c}) \leq \sum_{t}\sum_{l} \alpha_{k\prime t}^{l} x_{k\prime t}^{l} \leq U'_{k'} + M(1 - y_{k't}^{c}) \qquad \forall k' \in G, c \in C, t \in T \qquad (9)$$

$$L_{k\prime} - My_{k\prime t}^{c} \leq \sum_{t}\sum_{l} \alpha_{k\prime t}^{l} x_{k\prime t}^{l} \leq U_{k'} + My_{k\prime t}^{c} \qquad \forall k' \in G, c \in C, t \in T \qquad (10)$$

$$\frac{\sum_{k \in K} D_{kt} Pr_{kt}}{\sum_{k \in K} D_{kt}} \leq a \qquad \forall\, t \in T \qquad (11)$$

$$D_{kt} \times c_{kt}(p_{kt} - \sum_{l=1}^{r}\sum_{g} \alpha_{kt}^{l} x_{kt}^{l} + w_{kt}) \geq TP_{t} \qquad \forall k \in K, t \in T \qquad (12)$$

$$x_{kt}^{l},\ y_{kt}^{c} \in \{0, 1\}$$

Constraint (3) is our objective function is to maximize the sales paramount to the affected demand. Constraint (4) presents the fact that each product whether golden or ordinary, at each time should have only one promotion level. Constraint (5) indicates every item candidate for the golden discount should have the golden discount only once in all periods. Constraint (6) shows each period should have a certain number of golden products. Constraint (7) indicates we can only have a certain number of products as golden in each category in every period. Constraint (8) shows the discount calculation equation.. Constraint (9) expresses the promotion limitation for the golden products chosen for the period t. It is worth mentioning that those candidate golden products not chosen for the golden discount at the time t are treated as ordinary items. So, in Constraint (10) shows that each ordinary item's promotion should be in a determined upper and lower bound different from golden items. Constraints (11) formulated for the weighted average promotion consideration in each period for both golden and ordinary products. Constraints (12) indicate our profit should not be less than a determined amount.

5. Solution methods

5.1. Integer linear formulation

Linearization of constraint (11):

In Constraints (11) the binary variable x_{kt}^{l} in both D_{tk} and Pr_{tk} is multiplied by itself. So, we have to linearize it as below:

$$D_{tk}Pr_{tk} = (b_{tk} + \textstyle\sum_{l} \lambda_{ilk} b_{tk} \alpha_{kt}^{l} x_{kt}^{l} + \sum_{l}\sum_{q} \beta_{iklq} b_{tk} \alpha_{kt}^{l} x_{kt}^{l} + \sum_{l,t=1}^{t} \alpha_{kt}^{l} x_{kt}^{l} \theta_{k} \left(\frac{1}{2}\right)^{t-1}) \times \sum_{l} \alpha_{kt}^{l} x_{kt}^{l} \qquad (13)$$

Steps to Linearize the Constraint

1. Introduce Auxiliary Variables:

Let:

$$z_{ktl} = D_{kt} \times Pr_{kt} = D_{kt}(x_{kt}^{l} \alpha_{kt}^{l}) \qquad \forall k \in K, t \in T, l \in L \qquad (14)$$

This simplifies to:

$$z_{ktl} = D_{kt}(x_{kt}^{l} \alpha_{kt}^{l}) \qquad \forall k \in K, t \in T, l \in L \qquad (15)$$

2. Define x_{kt} and α_{kt}:

- x_{kt}: Binary variable indicating the selection of discount level.
- α_{kt}: Discount level (integer).

Linearize the Product $z_{ktl} = D_{kt}(x_{kt}^{l} \alpha_{kt}^{l})$:

Given that x_{kt}^{l} is binary, the product z_{ktl} can be linearized by considering the nature of binary multiplication:

$$z_{kt} = D_{kt} \times \alpha_{kt}^{l} \quad if\ x_{kt}^{l} = 1 \qquad \forall k \in K, t \in T \qquad (16)$$

$$z_{kt} = 0 \quad if\ x_{kt}^{l} = 1 \qquad \forall k \in K, t \in T \qquad (17)$$

3. Introduce Constraints to Linearize z_{kt}:

Using the property of binary variables:

$$z_{ktl} \leq M. x_{kt}^{l} \qquad \forall k \in K, t \in T, l \in L \qquad (18)$$

$$z_{ktl} \leq D_{kt}. \alpha_{kt}^{l} \qquad \forall k \in K, t \in T, l \in L \qquad (19)$$

$$z_{ktl} \geq D_{kt}. \alpha_{kt}^{l} - M. (1 - x_{kt}^{l}) \qquad \forall k \in K, t \in T, l \in L \qquad (20)$$

$$z_{ktl} \geq 0 \qquad \forall k \in K, t \in T, l \in L \qquad (21)$$

where M is a sufficiently large constant that bounds z_{ktl}. Typically, M can be set to the maximum possible value of $D_{kt}. \alpha_{kt}^{l}$.

4. Rewrite the Original Constraint:

Substitute z_{ktl} into the original constraint:

$$\frac{\sum_{k \in O} D_{kt} Pr_{kt}}{\sum_{k \in O} D_{kt}} \leq a \qquad \forall k \in O, t \in T \qquad (22)$$

becomes:

$$\sum_{k \in O} \sum_{l \in L} z_{ktl} \leq a \sum_{k \in O} D_{kt} \qquad \forall k \in O, t \in T, l \in L \qquad (23)$$

Linearization of constraints (12):

We have

$D_{kt} \times c_{kt}(p_{kt} - \sum_{l=1}^{r} \sum_{g} \alpha_{kt}^{l} x_{kt}^{l} + w_{kt})) \geq TP_t = D_{kt} \times c_{kt} \times p_{kt} - (D_{kt} \times c_{kt} \sum_{l=1}^{r} \sum_{g} \alpha_{kt}^{l} x_{kt}^{l}) + D_{kt} \times c_{kt} \times w_{kt} \geq TP_t$

So, we have:

$$-(D_{kt} \times c_{kt} \sum_{l=1}^{r} \sum_{g} \alpha_{kt}^{l} x_{kt}^{l}) \geq TP_t - D_{kt} \times c_{kt} \times p_{kt} - D_{kt} \times c_{kt} \times w_{kt} \qquad \forall k \in K, t \in T, l \in L \qquad (24)$$

So, we have:

$$\sum_{l=1}^{r} \sum_{g} D_{kt} \alpha_{kt}^{l} x_{kt}^{l} \leq \frac{TP_t - D_{kt} \times c_{kt} \times p_{kt} - D_{kt} \times c_{kt} \times w_{kt}}{-c_{kt}} \qquad \forall k \in K, t \in T, l \in L \qquad (25)$$

Assume a Positive variable $D_{kt} \alpha_{kt}^{l} x_{kt}^{l} = H_{klt}$

Therefore, we have the following inequalities:

$$H_{klt} \leq D_{kt} \alpha_{kt}^{l} \qquad \forall k \in K, t \in T, l \in L \qquad (26)$$

$$H_{klt} \leq x_{kt}^{l} \times M = \qquad \forall k \in K, t \in T, l \in L \qquad (27)$$

$$H_{klt} \geq D_{kt} \alpha_{kt}^{l} - ((1 - x_{kt}^{l}) \times M) \qquad \forall k \in K, t \in T, l \in L \qquad (28)$$

The mixed integer linear programming (MIP) model developed can be directly solved using MIP solvers. However, when dealing with large-scale problems, finding the global optimum within a practical time frame can be difficult.

5.2. Grey Wolf Algorithm

Grey Wolf Optimization (GWO) is a metaheuristic optimization algorithm that mimics the social structure and hunting strategies of grey wolves. This algorithm is favored for its simplicity, flexibility, ability to avoid local minima, and independence from gradient information (Kumar Chandar, S. 2020). The GWO algorithm simulates the leadership hierarchy among wolves, dividing them into alpha, beta, delta, and omega categories. During each iteration, the positions of wolves are updated based on their proximity to the prey (optimal solution), with the alpha wolf typically being the closest to the target.

This algorithm starts by initializing a population of wolves, where each wolf represents a potential solution. The wolves adjust their positions by moving towards the most promising solutions, simulating a hunting mechanism. Through repeated iterations, the wolves converge towards the best solution, which corresponds to the alpha wolf in the final iteration. How this method works is as follows:

1. Initialize search agents:

Let the search agents be denoted as S_i where $i = 1,2,3,\dots,n$ with V representing the number of decision variables and I_{max} as the maximum number of iterations.

2. Calculate vectors:

Compute vectors $\vec{L}$ and $\vec{P}$ using the following equations:

$$\vec{L} = 2\,.\,\vec{o}\,.\,q_1 - \vec{o}$$

$$\vec{P} = 2.\,q_2$$

where $\vec{o}$ decreases linearly from 2 to 0 over the iterations, and q_1 and q_2 are random numbers between 0 and 1.

3. Generate wolves:

Wolves are generated based on S and V as follows:

$$Wolves = \begin{bmatrix} w_1^1 & \dots & w_1^v \\ \vdots & \ddots & \vdots \\ w_s^1 & \dots & w_s^v \end{bmatrix}$$

4. Evaluation and Position Update

Evaluate the fitness of each agent:

Compute the difference between the current agent position and the prey's position $\vec{D}$:

$$\vec{D} = \left|\vec{P}.\vec{W}_{best}(t) - \vec{W}(t)\right|$$

Update the position:

$$\vec{W}(t+1) = \vec{W}_{best}(t) - \vec{L}.\vec{D}$$

5. Identify the best hunt agents:

Identify the best ($\vec{W_\alpha}$) , second-best ($\vec{W_\beta}$) and third-best ($\vec{W_\delta}$) hunt agents:

$$\vec{D_\alpha} = \left|\vec{P}_1.\vec{W_\alpha} - \vec{W}\right|$$

$$\vec{D_\beta} = \left|\vec{P}_1.\vec{W_\beta} - \vec{W}\right|$$

$$\vec{D_\delta} = \left|\vec{P}_1.\vec{W_\delta} - \vec{W}\right|$$

6. Update positions of the wolves:

Update positions based on the best hunt agents:

$$\vec{W_1} = \left|\vec{W_\alpha} - \vec{L}_1.\vec{D}_\alpha\right|$$

$$\overrightarrow{W_2} = |\overrightarrow{W_\beta} - \vec{L}_2.\vec{D}_\beta|$$

$$\overrightarrow{W_3} = |\overrightarrow{W_\delta} - \vec{L}_3.\vec{D}_\delta|$$

Final update:

$$\vec{W}(t+1) = \frac{\overrightarrow{W_1} + \overrightarrow{W_2} + \overrightarrow{W_3}}{3}$$

7. Iteration and Termination:

Iterate through the process:

For each iteration $i = 1\ to\ S$:

- Update the position of the current hunt agent.
- Recalculate the fitness of each hunt.
- Update the positions of $\overrightarrow{W_\alpha}$, $\overrightarrow{W_\beta}$, $\overrightarrow{W_\delta}$ based on fitness.

8. Check termination criteria:

If the maximum number of iterations I_{max} is reached, output $\overrightarrow{W_\alpha}$ Otherwise, repeat the steps.

Optimization algorithms often draw inspiration from physical phenomena, animal behavior, or evolutionary principles (Mirjalili et al., 2014). Among the most prominent and extensively used are PSO, GA, and ACO. A common limitation of many bio-inspired optimization algorithms is the absence of a consistent leader throughout the process. The Grey Wolf Optimization (GWO) algorithm addresses this issue by incorporating a natural leadership hierarchy observed in grey wolves. Introduced by Mirjalili et al. (2014), GWO is a type of Swarm Intelligence (SI) algorithm that replicates the hunting strategies and social order of grey wolves.

Grey wolves are part of the Canidae family and are known for their preference to live in packs, which are governed by a strict social structure. The leadership within the pack is typically held by a dominant pair, consisting of a male and a female wolf, known as the alphas (α) The alpha wolves are responsible for making critical decisions, such as those concerning hunting, resting periods, sleeping locations, and when to wake. Due to their authority, they are also referred to as the dominant wolves, with their commands being followed by the rest of the pack. Subordinate to the alphas are the betas (β), who can be either male or female. The betas assist the alphas in decision-making and help maintain order within the pack.

The omega (ω) wolves are at the lowest rank in the hierarchy and are required to submit to all the more dominant wolves. Any wolf that is neither an alpha nor a beta, nor an omega, is classified as a delta (δ). Delta wolves have authority over the omegas and report directly to the alphas and betas. Within the delta category, there are further roles, including elders, hunters, caretakers, and scouts.

6. Test Scenarios and Analysis

This section presents several examples to demonstrate the effectiveness of the proposed approaches. The scenarios explored include:

- Nonlinear Problem: The CONOPT solver, a Mixed-Integer Nonlinear Programming (MINLP) tool, is applied to tackle a nonlinear problem.
- Linearized Problem: The problem is linearized and then solved using the CBC solver, which is designed for Mixed-Integer Programming (MIP).
- GW Algorithm: The GW Algorithm is employed to solve the specified problem.

In each scenario, ten distinct problems were generated, with parameters randomly selected within the following ranges:

$b_{kt} \sim \mathrm{U}(50, 150), c_{kt} \sim \mathrm{U}(20,200), p_{kt} \sim \mathrm{U}(0.05, 0.5), \lambda_{kt}^{l} \sim \mathrm{U}(1,1.5), \beta_{kt}^{lq} \sim \mathrm{U}(1, 1.5), \theta_k\ \mathrm{U}(0, 0.2)$

$w_{kt} \sim \mathrm{U}(0,0.2), U_{kt} \sim \mathrm{U}(0.15, 0.25), U'_{k't} \sim \mathrm{U}(0.45,0.6), L_{kt} \sim \mathrm{U}(0, 0.08), L'_{k't} \sim \mathrm{U}(0.15, 0.25)$
$\alpha_{kt}^{l} \in \{0,0.1,0.2, \dots ,0.59,0.6\}\ TP_t \sim \mathrm{U}(800, 1600)$

Our initial step involves presenting detailed results for a selected configuration to demonstrate the process of generating subsequent summary tables. Table 1 displays the outcomes of 15 generated problems for the parameters t=6, o=20, g=5, c=5 and 80 runs of the GWA method, corresponding to the 15 generated problems. The last two rows show the average and standard deviation for the entire set. The percentage difference between the best solutions obtained by the GWA method and the optimal values determined by optimization solvers is presented in Table 1. The "Times Found" column indicates how often the GWA method identified the best solution. Additionally, the MINLP, MIP, and GWA methods were evaluated based on the CPU time spent solving the 15 generated problems.

Table 1
The differences in objectives and CPU time across ten instances with t=6, o=20, g=5, and c=5

Problem	Difference in objective value (%)	Count of identical responses	CPU seconds		
			CONOPT	CBC	GWA
1	0.217	2	1479.02	71.89	3.09
2	0.291	5	1341.7	80.57	2.83
3	0.973	3	1377.85	77.2	4
4	0.55	13	1207.13	74.78	3.38
5	0.201	13	1252.69	71.36	2.91
6	0.344	12	1421.03	80.86	3.04
7	0.353	8	1398.75	82.03	2.72
8	0.552	10	1479.61	73.98	4.28
9	0.978	7	1447.25	76.96	3.47
10	0.411	14	1204.46	72.88	3.25
11	0.12	12	1417.95	81.23	2.68
12	0.727	10	1221.27	78.19	2.88
13	0.824	13	1285.62	83	4.12
14	0.815	2	1358.88	82.52	4.78
15	0.513	0	1456.58	82.67	2.99
Average	**0.5242**	**8.266667**	**1356.653**	**78.008**	**3.361333**
Standard Deviation	**0.281611**	**4.787882**	**99.40518**	**4.180048**	**0.642127**

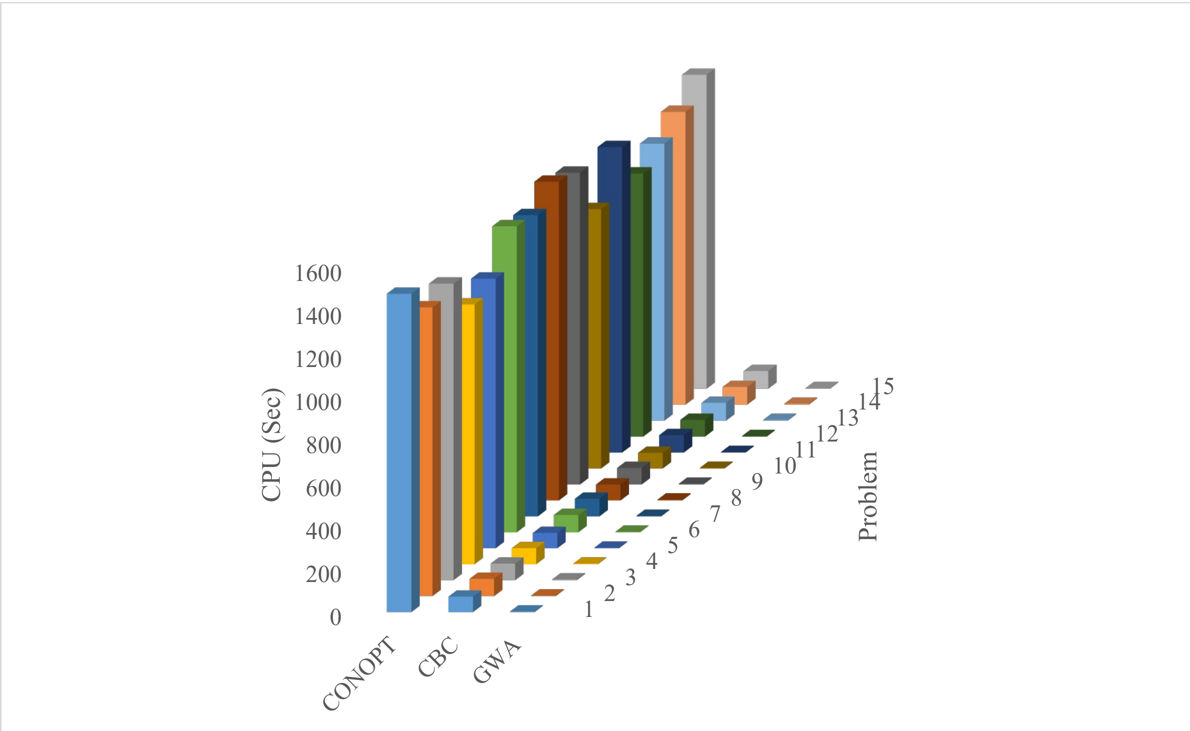

Fig. 1. Comparison of the three solution approaches in terms of computational time performance

Regarding the objective function, as shown in Table 1, the GWA method generally produces solutions that are close to those obtained by optimization solvers, though significant differences may arise in certain cases. Importantly, the GWA approach is considerably faster in execution compared to the other two methods. To simplify the presentation of the results, only average values will be provided in subsequent sections. A summary table will be organized by the number of products, with each row corresponding to a specific table, such as Table 2. These values will reflect the averages of the solved cases, with standard deviations noted in parentheses. To address the significant variations in objective function differences across the

10 problem instances, the maximum observed discrepancy will also be reported. Additionally, the final row will show the average difference across all configurations, independent of the product count in each setting.

Table 2
The mean and standard deviation for discrepancies in objective values and CPU time across scenarios with parameters set at t=6, o=20,40, g=5,10, and c=5

Products	Difference in value		CPU (seconds)		
	Obj (%)	Max (%)	CONOPT	CBC	GWA
25	0.52 (0.28)	0.98	1356.65 (99.40)	78.00 (4.18)	3.36 (0.64)
50	0.99 (0.68)	2.3	6745.75 (489.62)	105.55 (36.12)	4.26 (0.98)
All	**105.55 (36.12)**	**1.64**	**4051.2(294.51)**	**91.775(20.15)**	**3.81(0.81)**

Table 2 demonstrates the notable rise in solution time as the number of products increases, particularly when using the MINLP solver. Despite the increase, the GWA method not only remains extremely fast but also delivers high-quality solutions. Although the Mixed-Integer Programming (MIP) approach demands more CPU time than GWA, it ensures the optimal solution while keeping the processing time relatively brief. A detailed analysis of this trend is provided in Table 3, where the number of product Groups and products has been expanded.

Table 3
The average and standard deviation values for the differences in objective functions and CPU time across cases with c=5,10,20,30 and o=40,80,160,240,320, and g=10,20,40,60,80 and t=6.

Product Groups	Products	Changes in		CPU seconds		
		Objective (%)	Maximum (%)	CONOPT	CBC	GWA
5	50	0.52 (0.28)	0.98	1356.65 (99.40)	78.00 (4.18)	3.36 (0.64)
	100	0.78(0.26)	1.42	8561(2145)	150(42)	4.56(1.35)
10	100	1.12(0.38)	1.85	-	316(85)	8.36(2.32)
	200	1.36(0.34)	2.36	-	1021(289)	20.45(4.96)
20	200	1.85(0.46)	3.85	-	2938(863)	45.27(10.24)
	300	2.45(0.63)	7.2	-	6993(1547)	60.87(16.87)
30	400	3.35(0.9)	15.1	-	21074(4523)	125.63(36.52)
All		1.63 (0.46)	4.68	-	4652 (1050)	38.35 (10.4)

Table 3 shows that with a slight increase in the number of product types and products, CPU processing time for the MINLP solver significantly increases, while the MIP solver and heuristic methods are less affected. This scale indicates that using the MINLP solver is impractical, and the MIP solver and heuristic methods should be used instead. Due to shorter processing times, heuristic methods are more suitable for larger problems, while the MIP solver remains efficient for medium-sized problems. The results indicate that CPU time for the MIP solver increases significantly with problem size, but the GWA method shows less sensitivity to these changes.

In the final stage, the quality of the GWA solution was evaluated regardless of problem size. In this study, a set of test cases with diverse characteristics was used to statistically assess the effectiveness of the GWA method. A total of 15 problems with varying sizes were created, and each problem was executed 80 times using the GWA algorithm. The results are summarized in Table 4.

Table 4
The results for 15 Problems that Randomly Generated

Case	Product Types	Products	Difference in obj (%)
1	5	50	0.12
2	5	75	0.51
3	5	100	1.36
4	10	100	1.23
5	10	150	0.87
6	10	200	1.35
7	20	200	1.6
8	20	250	0.25
9	20	300	1.38
10	30	300	1.6
11	30	350	1.3
12	30	400	0.87
13	40	400	1.23
14	50	450	1.85
15	60	500	1.93
Average			**1.163**
SD			**0.54**

To assess the performance of the GW algorithm, a t-test (hypothesis testing) is conducted. In this study, the term "effectiveness" is used to describe the algorithm's ability to consistently produce high-quality solutions. As shown in Table 4, the term "Difference in object" and serves as a metric for how close a result is to the global optimum.

The purpose of this effectiveness test is to determine whether the null hypothesis $H_0: \mu_{Diff} \leq 1\%$can be rejected in favor of the alternative hypothesis $H_1: \mu_{Diff} > 1\%$ This test evaluates whether the quality of the solutions obtained exceeds 98%. Eq. (38) illustrates the calculation of the t-value:

$$t = \frac{\overline{Diff} - 1}{S(Diff)/\sqrt{n}} \tag{38}$$

To verify if the data follows a normal distribution, the Kolmogorov-Smirnov (K-S) test is applied. The resulting test statistic (D) yields a P-value of 0.92, indicating no significant deviation from normality. Additionally, the t-test produces a P-value of 0.81, suggesting that the GWA algorithm is effective in delivering high-quality solutions.

6.1. Case Study

This section focuses on implementing an optimal product assortment and promotion model for a chain store in Tehran, Iran. We conducted research for the "Ofogh Koorosh" chain stores located in Tehran. For the implementation of this model in the store, based on the organization's policies and business managers' guidelines, it was agreed that the golden products would be identified by the marketing team and provided as a parameter to the model according to the organization's policies. Given this organization's policy, the variable y_{kt}^{c} is no longer binary and is instead treated as a parameter within the model. Under these conditions, the model can be approximated as linear.

We can consider the promotion values as a continuous variable between zero and one. In other words, instead of using specific discount values defined as $\sum_l \alpha_{kt}^{l} x_{kt}^{l}$, we can treat x_{kt} as a continuous variable. With the current conditions where y_{kt}^{c} is a parameter and x_{kt} is treated as linear, all constraints remain linear according to the specified linearization method, except for constraint 14, which requires linearization.

To linearize constraint 14, we Have:

$$D_{kt} \times c_{kt}(p_{kt} - x_{kt} + w_{kt}) \geq TP_t$$

So, we have:

$$(D_{kt} \times c_{kt} \times p_{kt}) - (D_{kt} \times c_{kt} \times x_{kt}) + (D_{kt} \times c_{kt} \times w_{kt}) \geq TP_t$$

The only nonlinear term in the above constraint is $D_{kt} \times x_{kt}$ To linearize the constraint, it is sufficient to rewrite the term $D_{kt} \times x_{kt}$ in a linear form. To do this, we first need to substitute the value of D_{kt} into the constraint. In doing so, we obtain the following:

$$D_{kt} \times x_{kt} = (b_{kt} + \lambda_{kt}^{l} b_{kt} \alpha_{kt}^{l} x_{kt} + \sum_{q} \beta_{kt}^{lq} b_{kt} x_{qt} + \sum_{t=1}^{t} x_{kt} \theta_k \left(\frac{1}{2}\right)^{t-1}) \times x_{kt}$$

$$= b_{kt} \times x_{kt} + \lambda_{kt}^{l} b_{kt} \alpha_{kt}^{l} {x_{kt}}^2 + \sum_{q} \beta_{kt}^{lq} b_{kt} x_{qt} x_{kt} + \sum_{t=1}^{t} {x_{kt}}^2 \theta_k \left(\frac{1}{2}\right)^{t-1}$$

In the above constraint, the terms ${x_{kt}}^2$ *and* $x_{qt}x_{kt}$ are responsible for the nonlinearity of the model. To address this, it is sufficient to replace these terms with linear expressions. Considering the advancements in industry, increased competitiveness in the retail sector, and product profit margins, studies indicate that discounts typically range between 10% and 40%. This discount range not only creates sufficient appeal for customers but also reasonably maintains the profit margins of stores. Additionally, the model's results confirm this pattern, showing that most discounts applied in stores fall within this range, acting as a balance point between attracting customers and maintaining profitability.

Now, assuming x_{kt} falls within this range, the terms involving ${x_{kt}}^2$ *and* $x_{qt}x_{kt}$ can be linearized using Taylor expansion (Groza al. 2013).

Linearization of the Function $x_{qt}x_{kt}$:

To linearize the function $x_{qt}x_{kt}$ using Taylor series expansion, we must follow these steps:

1. Function definition and expansion point selection

We start with the function $x_{qt}x_{kt}$ To linearize this function, we need to select an expansion point (also called the reference point). Typically, we choose the midpoint of the given ranges for x_{qt} and x_{kt}. According to our assumptions about discount amounts the ranges are between 0.1 and 0.4. Thus, the expansion point is chosen as:

x_{kte}=x_{qte}=0.1+0.42=0.25

2. Expansion Taylor series in the reference point

The first-order Taylor series expansion of the function $x_{qt}x_{kt}$ around the point x_{kte},x_{qte} is given by:

$$x_{qt}x_{kt} \approx x_{kte}x_{qte} + \left(\left(\frac{\partial\, x_{qt}x_{kt}}{\partial x_{kt}}\Big|\left(x_{kte}, x_{qte}\right)\right) \times (x_{kt} - x_{kte})\right) + \left(\left(\frac{\partial\, x_{qt}x_{kt}}{\partial x_{qt}}\Big|\left(x_{kte}, x_{qte}\right)\right) \times (\, x_{qt} - x_{qte})\right)$$

3. Calculation of partial derivatives and placement in the expansion function

After taking the derivative and substituting it into the function, we have:

$$x_{qt}x_{kt} \approx 0.25x_{kt} + 0.25\, x_{qt} - 0.0625$$

Linearization of the Function ${x_{kt}}^2$:

In accordance with the method outlined for the function $x_{qt}x_{kt}$, we apply the same steps to the function ${x_{kt}}^2$, resulting in:

$${x_{kt}}^2 \approx 0.5x_{kt} - 0.0625$$

By substituting the two expressions ${x_{kt}}^2$ and $x_{qt}x_{kt}$ into the model, it will be linearized. This means that when, according to the organization's policies, the Golden product is to be determined by the organization itself, the model can be linearized with a good approximation by considering continuous variables for the discount rate. Linearizing the model allows the problem to be solved on a larger scale in significantly less time compared to the MIP model. To assess the efficiency of the linearization method, we conducted 20 case studies, each time on 80 to 120 different products across 5 branches of Ofogh Kourosh stores in the Mehrabad region. We compared the optimal solution of the approximated function with the MIP function, and the results are presented in Table 5.

Table 5

The results for comparing the optimal solutions of the two models, LP and MIP, based on a case study involving 80 to 120 products across 5 stores in the Mehrabad region

Case	Products	CPU (seconds)		Difference in obj (%)
		MIP	LP	
1	80	323.2	3.35	1.4
2	80	310.7	3.5	1.05
3	80	311.9	3.34	1.3
4	80	345.2	3.46	0.43
5	90	384.9	3.41	1.01
6	90	375.2	3.27	0.6
7	90	414.4	3.31	1.04
8	90	428.4	3.3	0.65
9	100	444.8	3.77	1.35
10	100	438.5	3.48	0.45
11	100	441.1	3.7	1.22
12	100	433.3	3.4	0.53
13	110	445.4	3.93	0.42
14	110	463.6	3.63	1.12
15	110	482.4	3.75	1.45
16	110	496.5	3.61	1.89
17	120	487.6	3.63	0.88
18	120	499.1	3.92	0.53
19	120	501.2	3.97	0.69
20	120	505.2	3.8	1.91
Average				0.99

The analysis indicates that the LP model delivers an optimal solution with a remarkably close approximation to the MIP model, maintaining Difference in obj function of less than one percent. Despite this high level of accuracy, the LP model requires significantly less computation time compared to the MIP model. This reduced time complexity makes the LP model particularly advantageous for solving large-scale problems, such as scenarios involving more than 3,000 different SKUs across approximately 3,000 retail stores.

In large-scale applications, where computation time and resource efficiency are critical, the LP model demonstrates its superiority by delivering results faster without compromising on the quality of the solution. This efficiency not only accelerates decision-making processes but also reduces computational costs, making it a practical choice for extensive retail operations.

Moreover, when compared to the GWA method, the LP model not only yields solutions that are closer to the optimal objective function but also offers greater simplicity in implementation. The straightforward nature of the LP model's formulation, coupled with its relatively shorter solution time, provides an edge in practical scenarios where ease of use and rapid execution are crucial. This makes the LP model a more accessible and efficient tool for practitioners who need to balance accuracy, computational resources, and ease of deployment in large-scale optimization problems.

Due that the model has been effectively linearized, we proceed to implement it across the Ofogh Koorosh chain stores in three major cities of Tehran, Mashhad, and Isfahan. To accurately estimate the demand in each of these cities, we utilized advanced machine learning techniques on the comprehensive sales data collected from the stores over the past several months. This study encompasses the evaluation of approximately 3000 different products across six distinct time periods, each period consisting of one week. Within each of these periods, around 600 products were strategically selected as Golden products by the marketing team based on their significance and potential for boosting sales. The primary objective of this model implementation is not only to maximize demand and consequently capture a larger share of the market but also to ensure that the minimum required profit margin from product sales is maintained. By deploying this model, we aim to provide precise guidance on the optimal discount rates that should be applied to each product in every time period, thereby enhancing the overall sales performance and competitiveness of the stores in these cities.

6.2. Model Results

Given the high volume of data in the case study, we focus solely on presenting the results of the model. The optimal discount rates for Golden products range from 25% to 40%, while the optimal discount rates for other products generally fall within the range of 10% to 15%. After implementing the model in the store, the expected profit from sales was achieved, and the sales volume increased by 7.68% compared to the store's expected sales. This figure represents a significant increase, considering the store's sales volume.

6.3. Evaluation of Model Performance

To further evaluate the performance of the model in real-world conditions, a three-week experiment was conducted focusing on three categories of products, including dairy products, semi-prepared foods, and sausages. In this experiment, one store was selected as a pilot site for implementing the model. To account for potential variations in demand between the evaluation period and the previous period due to other factors, a similar store (which exhibits comparable sales behavior) was chosen to continue the previous status for the aforementioned products. This approach allowed for a more effective evaluation of the model's performance.

Table 6
Sales and profits of Pilot store and similar store for three product categories of semi-prepared and sausages and dairy in two three-week intervals

Category	Time	Sales Volume		Profit Value	
		Pilot	Similar	Pilot	Similar
semi-prepared	3 Week Before Implementation of Model	514	520	5987	6051
sausages		638	641	4583	4600
dairy		236	228	2146	2080
Total		1388	1389	12716	12731
semi-prepared	3 Week Of implementation of Model	536	501	5812	5832
sausages		678	611	4469	4395
dairy		245	231	2030	2100
Total		1459	1343	12311	12327
% Change		5.12%	-3.31%	-3.18%	-3.17%

According to Table 6, during the second time period, the pilot store experienced an increase in sales accompanied by a decrease in profit. However, it is noteworthy that the target profit margin for these products was achieved in the pilot store. In contrast, the similar store observed both a decline in sales and a decrease in profit. These results suggest that the implementation of the model can effectively capture a significant share of the retail market, while ensuring that the desired profit margins are met and the overall profitability of the store remains intact.

7. Conclusion

In today's highly competitive retail environment, the development and implementation of effective pricing strategies are crucial for maintaining profitability and capturing market share. This study introduces a novel hybrid optimization model that strategically combines the Everyday Low Pricing (EDLP) and High-Low (HL) pricing strategies. By leveraging the strengths of both approaches, the model offers a comprehensive solution to the complexities of dynamic retail markets, addressing the challenges of demand fluctuations, inventory management, and profitability. The model was developed in collaboration with a major Iranian supermarket chain, allowing for the integration of real-world data into the optimization process. Through extensive numerical experiments, the model demonstrated its ability to increase sales while maintaining profitability within acceptable bounds, proving its practical applicability in large-scale retail operations. The use of the Grey Wolf Optimization (GWO) algorithm further enhanced the model's computational efficiency, making it a robust tool for decision-making in complex retail environments.

This research makes several significant contributions to the field of retail pricing strategies. Firstly, it offers a scalable and practical solution for retailers seeking to optimize their pricing strategies in a data-driven manner. The linearization of the model for predetermined "Golden" products simplifies the computational process, enabling its application in larger retail settings. Secondly, the integration of EDLP and HL strategies within a single model provides retailers with the flexibility to adapt their pricing approaches based on market conditions and customer behavior, thereby maximizing their competitive advantage. The findings of this study have important implications for both academia and industry. For researchers, the hybrid model opens new avenues for exploring the interplay between different pricing strategies in various retail contexts. For practitioners, the model offers actionable insights and a strategic framework for optimizing pricing decisions, ultimately supporting long-term business sustainability and market dominance.

Given the growing complexity of retail markets and the increasing reliance on data-driven decision-making, the hybrid pricing model presented in this study is well-positioned to meet the evolving needs of modern retailers. Future research could explore further refinements to the model, such as incorporating more sophisticated demand forecasting techniques or expanding its application to other sectors within retail. Nevertheless, this study represents a significant step forward in the optimization of retail pricing strategies, providing a valuable tool for retailers to navigate the challenges of today's dynamic market landscape.

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