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Pricing decisions in a closed loop supply chain with focus preference under the carbon trading scheme

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A B S T R A C T

This paper investigates a closed loop supply chain (CLSC) encompassing a manufacturer, a retailer, and consumers operating within the carbon trading scheme. Employing the focus theory of choice, we analyze the decision-making processes of the retailer, considering various personality traits. A Stackelberg game is formulated, wherein the manufacturer assumes responsibility for recycling activities. The research explores the impact of the retailer's optimism and confidence levels on optimal decision-making within a positive evaluation system. Numerical examples are employed to elucidate equilibrium solutions, illustrating the correlation between the retailer's personality traits and the manufacturer's optimal decisions. Furthermore, a sensitivity analysis is conducted on the carbon trading price and the manufacturer's carbon emission quota allocation within a single cycle under the carbon trading scheme. The investigation concludes with an examination of the influence of recycling prices on the manufacturer's optimal revenue. The findings indicate that retailers with distinct personality traits adopt varied pricing strategies. Decreases in optimism and self-confidence levels prompt the retailer to opt for relatively lower retail profit pricing. Simultaneously, the manufacturer demonstrates a preference for collaborating with a retailer characterized by optimism and lower confidence levels, thereby enhancing overall manufacturing revenue. Notably, under the carbon trading scheme, fluctuations in carbon trading and recycling prices distinctly influence the manufacturer's decisions.

1. Introduction

The contemporary landscape of economic and societal advancements has witnessed a significant surge in product retention within the market, accompanied by escalating concerns pertaining to excess resource utilization and environmental pollution (Wang et al., 2020). In response to these challenges, there is a growing consensus that economic recovery must be aligned with principles of environmental sustainability, emphasizing the imperative of fostering a green and low-carbon trajectory to address climate change and ecological crises. Notably, China has proactively endorsed the "carbon peaking and carbon neutrality" policy in recent years, aiming to propagate a lifestyle that is not only green and environmentally friendly but also low-carbon. This strategic initiative serves as a catalyst for steering green technological innovation and enhancing the global competitiveness of industries and economies. A coherent policy framework conducive to the realization of "carbon peaking and carbon neutrality" is indispensable for attaining dual carbon goals. Such a framework facilitates a transition from resource-dependent paradigms to technology-driven solutions and concurrently supports the transformation of policy systems aligned with technological innovation. This nuanced approach reflects a pivotal shift in the overarching policy landscape, emphasizing the integral role of technology in shaping sustainable practices and fostering a harmonious coexistence between economic development and environmental preservation. The escalating challenge of global warming, attributed to carbon emissions, has emerged as a preeminent non-traditional security concern, posing formidable constraints on the sustainable development of societies. Addressing this concern, the implementation of a carbon emission trading system has proven effective in curbing emissions by optimizing resource allocation through market mechanisms (Mei et al., 2022). Beyond the pivotal realms of

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industry and energy restructuring, various meaningful measures are instrumental in achieving low-carbon emissions, thereby reconciling economic development with environmentally conscious transformation. A particularly impactful strategy is the management of used products through recycling, aiming to restore maximum monetary value with minimal costs while adhering to technical, ecological, and legal considerations—termed as closed loop supply chain (CLSC) management. Recognized as a sustainable developmental approach (Hong et al., 2013), the management of CLSC is garnering increased attention from companies in response to mounting environmental pressures. Such a strategic framework contributes to both emission reduction and ecological preservation while simultaneously yielding significant reductions in production costs for enterprises (Bendoly et al., 2006). In the contemporary landscape, major electronic product manufacturers are actively engaged in recycling initiatives through diverse channels, both online and offline, to recover waste electronic products from consumers. Illustratively, Apple has initiated the "Apple Trade In" program, while Huawei has similarly introduced a cell phone recycling business. These initiatives underscore the industry's commitment to CLSC practices as a pivotal component of their environmental sustainability endeavors.

The carbon trading scheme significantly influences the operational aspects of enterprises and their decisions regarding emission reduction strategies. By endowing carbon dioxide emission rights with commodity attributes, the carbon trading system facilitates the unrestricted buying and selling of these rights in the market within the confines of allocated carbon quotas. Building upon this framework, Ma et al. (2014) established a model to derive target carbon emissions and optimal prices for low-carbon products within the production cycle. Toptal et al. (2014) explored the environmental sensitivity of consumers and concluded that the carbon allowance trading policy, when coupled with emission reduction investments, can effectively lower the carbon emissions and costs incurred by enterprises. Yang et al. (2017) incorporated carbon emissions considerations and provided enterprises with the most effective pricing approaches and investment plans for reducing emissions tailored to different channels. Considering consumers' preferences for low-carbon products and channel preferences, Sun et al. (2018) comprehensively identified the optimal carbon emission reduction boundary through comparative decision models. Pang et al. (2018) delved into the impact of carbon allowance trading scheme prices and consumers' low-carbon preferences on the carbon emissions of diverse manufacturer types operating under carbon allowance trading policies. This body of research collectively contributes to a nuanced understanding of the complex interactions among carbon trading regimes, consumer preferences, and enterprises' emission reduction strategies within the broader context of sustainability. The predominant focus in existing research on CLSC lies in the cooperative model of upstream and downstream enterprises, characterized by bounded rationality. This entails the examination of game models between different decision-makers within the supply chain. Traditional research in this domain typically characterizes decision-makers as "economic man", assuming full rationality and employing "expected utility maximization" as the guiding decision criterion. Noteworthy economists such as von Neumann and Morgenstern (2007) introduced the expected utility theory in 1944, followed by Savage's proposition of the subjective expected utility theory (Mahalanobis, 1954). Despite theoretical underpinnings, empirical evidence consistently indicates systematic deviations from these axioms (Beach & Lipshitz, 2017; Cachon & Lariviere, 2005; Katok & Pavlov, 2013; List & Mason, 2009; Loch & Wu, 2007; Tocher, 1977). The burgeoning fields of behavioral economics and psychology underscore the significance of behavioral preferences in shaping decisions within supply chain dynamics. Experimental findings suggest that salience information is a pivotal determinant in human decision-making (Brandstätter & Korner, 2014; Busse et al., 2013; Lacetera et al., 2012; Qiu et al., 2022). Guo (2010a, 2010b, 2011) introduced the notion that individuals assess decision alternatives based on associated events, that is, each individual has a more concerned event, called the "decision focus". This event is particularly prominent to the individual due to the resulting payoff and probability. This conceptualization led to the development of the one-shot decision theory. Building upon axiomatization, Guo (2019) further refined this theory by proposing the focus theory of choice, utilizing analytical and mathematical tools. Subsequently, Zhu et al. (2023) applied this theory to reframe the analysis of the newsvendor problem, wherein the optimal order quantity is determined by the focus of the order quantity rather than the expected utility. This evolving paradigm in decision theory and supply chain research highlights the increasing recognition of the role of behavioral factors and salience information in influencing decision-making processes.

This paper delves into the analysis of retailers' behavior within a manufacturer-dominated CLSC model. A novel CLSC model is formulated by integrating the positive evaluation system of the focus theory of choice. Subsequent numerical analyses elucidate the intricate relationships between retailers' optimism and confidence levels and their consequential impact on optimal retail profit pricing, the manufacturer's wholesale price, and overall revenue. Furthermore, an exploration into the impact of carbon trading and recycling prices on manufacturers' decisions within the carbon trading scheme is conducted. The findings underscore that diminishing levels of retailer optimism and self-confidence correlate with a tendency toward lower retail profit pricing. This strategic pricing approach not only stimulates market demand but also enhances the likelihood of favorable outcomes. The research reveals a manufacturer's inclination to collaborate with retailers characterized by higher optimism and lower confidence levels. Within the carbon trading scheme, the prevailing carbon trading price directly influences the wholesale prices of the manufacturer. In instances of higher carbon trading prices, the manufacturer may adjust wholesale prices to modulate market demand, consequently curbing production and carbon emissions. Conversely, excessively elevated carbon trading prices may prompt the manufacturer to sell surplus carbon credits in the market, generating additional revenue while regulating carbon emissions. Furthermore, the choice of recycling price emerges as a pivotal factor impacting wholesale prices, retail pricing, and the manufacturer's expected revenue. A discernible pattern emerges, where an increase in recycling price corresponds to a fall in wholesale prices, an increase in retail profit, and an initial rise followed by a decline in the manufacturer's expected revenue. This nuanced analysis emphasizes the critical importance of selecting an optimal recycling price, as lower recycling prices result in reduced recycling, leading to diminished savings in remanufacturing and carbon reduction costs. The subsequent sections of this paper are outlined below. Section 2 presents the CLSC model within the context of the carbon trading scheme, providing a comprehensive overview, while concurrently reviewing the classical model grounded in expected value utility. Using the focus theory of choice, Section 3 elucidates the supply chain decision model. This section examines the retailer's decision-making process, incorporating behavioral preferences, and derives optimal solutions for the retailer across various personality traits. Section 4 proceeds to conduct numerical experiments, facilitating the determination of equilibrium solutions through illustrative examples, and systematically delineates the impact of personality traits on the optimal decisions made by the manufacturer. Additionally, this section scrutinizes the effects of carbon trading price, manufacturers' carbon emission credits within a single cycle, and recycling prices on the decisions made by manufacturers within the framework of the carbon trading scheme. A comprehensive analysis is provided, complemented by managerial insights. Finally, Section 5 draws conclusions and offers closing remarks, summarizing the key findings and contributions of our study to the field of CLSC management under the influence of the carbon trading scheme. To improve the readability of this paper, all proofs are provided in Appendix.

2. Closed loop chain model

2.1 Model description and basic assumptions

This section examines a CLSC comprising a manufacturer and a retailer. The manufacturer is actively involved in the recycling of waste products from the preceding cycle and the subsequent remanufacturing of products. Specifically, the quantity of products received by the manufacturer from consumers is denoted as Q. It is noteworthy that Q is influenced by both the minimum recycling quantity, denoted as n, and the recycling price, denoted as b, represented by the equation $Q = n + \alpha b$, where $\alpha > 0$ signifies consumers' sensitivity to the recycling price. In its role as a producer, the manufacturer utilizes raw materials and recycled products for the production of new items. The cost associated with employing recovered products is denoted as c_o. In cases where the quantity of recycling falls below market demand, the manufacturer processes partial products using raw materials, incurring a cost denoted as c_n. Following the production process, the manufacturer proceeds to sell the products to the retailer at a price denoted as w. Subsequently, the product is sold by the retailer to the consumer at the price of r, aiming to generate retail profits denoted as δ, such that $r = w + \delta$. This configuration reflects the interaction of various factors within the CLSC, encompassing recycling, remanufacturing, and pricing strategies adopted by the manufacturer and retailer to meet market demands and maximize profitability. Fig. 1 depicts the CLSC structure.

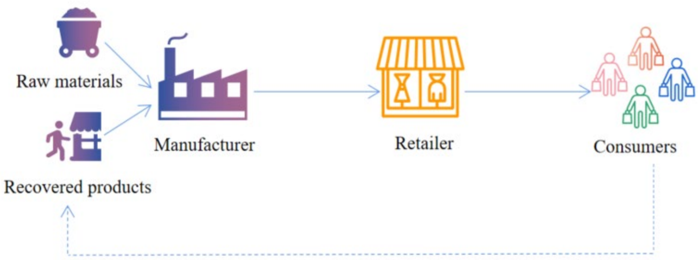

Fig. 1. The CLSC structure

Let us posit that the unit cost of products manufactured from raw materials surpasses that of remanufactured products, symbolically expressed as $c_n > c_o$. Additionally, the unit carbon emission associated with products produced from new materials exceeds that of remanufactured products, represented as $e_n > e_o$. It is also assumed that the transaction prices for manufacturers engaging in the purchase or sale of carbon emission rights within the carbon trading market are equivalent. The relevant notations are detailed in Table 1 for clarity and reference. This delineation establishes a set of foundational assumptions, essential for the subsequent analytical framework and modeling considerations within the context of the CLSC.

Table 1
Notations

Variable	Explanation
w	Unit wholesale price of the manufacturer (USD)
δ	Unit retail profit of the retailer (USD)
Parameter	**Explanation**
b	Unit recovery price of products ($b > 0$) (USD)
α	Sensitivity coefficient of consumers for recycling prices
β	Sensitivity coefficient of consumers for retail prices
D	Market demand of products
X	Maximum potential market demand
Q	Number of recycled products
n	Minimum recycling volume in the market
E	Carbon emission credits within a single cycle
P	Trading price of unit carbon emission rights (USD)
s	Unit residual value of scrap products (USD)

Within the framework of a wholesale price contract, the dominant manufacturer typically assumes the initiative by stipulating the wholesale price w, thereby influencing the retailer to determine the retail profit δ. Diverse retail prices have consequential effects on the market demand D, wherein the market demand is contingent upon the maximum potential market demand X, the retail profit δ, and the consumer's demand price sensitivity coefficient β. Mathematically, this relationship is expressed as $D = X - \beta(\delta + w)$. Consequently, both of them engage in a Stackelberg game, with the manufacturer functioning as the leader setting the wholesale price and the retailer as the follower determining the retail profit.

The problem is predicated on the premise of "recycling before manufacturing" due to the inherent randomness of market demand. Given the unpredictable nature of recycling and market demand, akin to the challenges posed by demand fluctuations in inventory decision problems, two scenarios necessitate consideration:

1) When the recovery quantity is less than the subsequent cycle market demand, the manufacturer invests additional raw materials to produce more products, aligning with the target market demand.
2) When the recovery quantity surpasses the subsequent cycle market demand, the excess recovered products beyond the market demand can be disposed of at salvage value without the need for further raw material investment.

Consequently, the profit function for the manufacturer is articulated as

$$v_m(x,w) = \begin{cases} wD - c_n(D-Q) - (c_o+b)Q + P(E - e_oQ - e_n(D-Q)), & Q \le D, \\ (w-c_o)D + s(Q-D) - bQ + P(E - e_oD), & Q > D. \end{cases} \quad (1)$$

In the scenario where $Q \le D$, denoting a recovery quantity less than market demand, wD signifies sales revenue, $c_n(D-Q)$ signifies production costs with raw materials, $(c_o+b)Q$ signifies costs associated with recovered products, and $P(E - e_oQ - e_n(D-Q))$ signifies costs of carbon quota trading in the corresponding market. In the situation where $Q > D$, representing a recovery quantity exceeding market demand, wD signifies sales revenue, c_oQ signifies costs of recovered products, $(Q - D)$ represents profits from the disposal of residual value, bQ signifies recycling costs, and $P(E - e_oD)$ represents costs of carbon quota trading in the carbon trading market. The profit function for the retailer is succinctly expressed as

$$v_r(x,\delta) = \delta(x - \beta(\delta + w)),$$

where δ and $x - \beta(\delta + w)$ denote the retail profit and market demand, respectively. This comprehensive representation captures the intricate dynamics of profit maximization within the CLSC.

2.2 Expectation-based supply chain model

In traditional supply chains, decision-makers typically adopt the maximization of expected utility as their primary decision criterion. This section initiates by providing a comprehensive review of how both retailers and manufacturers make decisions within a CLCS framework, grounded in traditional expectations. Considering a known probability density function $f(\cdot)$ and cumulative distribution function $F(\cdot)$ characterizing the maximum potential market demand, the following conditions are assumed to be satisfied:

a) The potential market demand is confined to the interval $\Omega = [l,h]$;
b) $f(x) > 0$ is strictly quasi-concave, and there exists $x_0 \in [l,h]$ such that

$$f(x_0) = \max_{x\in[l,h]} f(x).$$

Let m denote the expected value of the random demand X. The retailer's expected return is

$$G(\delta) = E\big(v_r(X,\delta)\big) = \delta\big(m - \beta(\delta + w)\big).$$

The first and second derivatives of $G(\cdot)$ are computed as follows:

$$G'(\delta) = m - 2\beta\delta - \beta w,$$
$$G''(\delta) = -2\beta < 0.$$

It is evident from the concavity of $G(\cdot)$ as indicated by $G''(\cdot)$ being negative, that the function achieves its maximum value when its first derivative is zero. Consequently, to maximize revenue, the unit retail profit for the retailer δ^* is determined by the equation

$$\delta^* = \frac{m - \beta w}{2\beta}. \quad (2)$$

This analytical expression provides a strategic insight into the optimal unit retail profit that facilitates revenue maximization for the retailer, subject to the operational confines of supply chain. Upon elucidating the retailer's behavior, the focus shifts to the decision-making process for the supplier. In accordance with (1), the expected revenue for the supplier is formulated as

$$E\left(v_m(X,w)\right) = \int_{n+\alpha b+\beta(\delta+w)}^{h} \begin{pmatrix} w\left(x-\beta(\delta+w)\right)-c_n\left(\left(x-\beta(\delta+w)\right)-(n+\alpha b)\right)-(c_o+b)(n+\alpha b) \\ +P\left(E-e_o(n+\alpha b)-e_n\left(\left(x-\beta(\delta+w)\right)-(n+\alpha b)\right)\right) \end{pmatrix} f(x)dx + \int_{l}^{n+\alpha b+\beta(\delta+w)} \begin{pmatrix} (w-c_o)\left(x-\beta(\delta+w)\right)+s\left((n+\alpha b)-\left(x-\beta(\delta+w)\right)\right)-b(n+\alpha b) \\ +P\left(E-e_o\left(x-\beta(\delta+w)\right)\right) \end{pmatrix} f(x)dx \tag{3}$$

By substituting (2) into (3), the expected revenue for the manufacturer is derived as

$$\begin{aligned} M(w) &= -\left(w-c_n-\tilde{P}e_n\right)\int_{n+\alpha b+\beta\left(\frac{m-\beta w}{2\beta}+w\right)}^{h} F(x)dx + (-w+c_n+Pe_n)\beta\left(\frac{m-\beta w}{2\beta}+w\right) \\ &\quad -(w-c_o-s-Pe_o)\int_{l}^{n+\alpha b+\beta\left(\frac{m-\beta w}{2\beta}+w\right)} F(x)dx + (c_n-c_o-b-Pe_o+Pe_n)(n+\alpha b) \\ &\quad +PE+h(w-c_n-Pe_n). \end{aligned}$$

The first-order derivative of the expected revenue for the manufacturer is expressed as:

$$\begin{aligned} M'(w) = -\int_{l}^{h} F(x)dx + \frac{\beta}{2}F\left(n+\alpha b+\beta\left(\frac{m}{2\beta}+\frac{w}{2}\right)\right)(c_o+s+Pe_o-c_n-Pe_n) \\ -(w-c_n-Pe_n)-\beta\left(\frac{m}{2\beta}+\frac{w}{2}\right)+\frac{\beta}{2}(-w+c_n+Pe_n)+h. \end{aligned}$$

The optimal wholesale price w^* set by the manufacturer satisfies $M'(w^*)=0$. Furthermore, this facilitates the derivation of the manufacturer's optimal expected revenue $M(w^*)$ and the optimal retail profit δ^*. This analytical framework establishes a rigorous foundation for optimizing the supplier's decision-making process within the CLSC. The subsequent analysis is presented through an illustrative example with specified parameters. The parameters are defined as follows: the manufacturer's unit production costs $c_n=200,\ c_o=120$, unit recovery price $b=30$, and unit residual value $s=20$. The consumers' sensitivity coefficient to price $\beta=5$, the consumers' coefficient sensitivity to the recycling price $\alpha=20$, and the minimum recycling volume $n=100$. Government-mandated carbon credits for manufacturers in a single cycle $E=4200$, carbon emissions per unit of product $e_n=7, e_o=5$, and the carbon credits per unit price in the carbon trading market $P=5$. The market demand X spans the interval $[4200,5000]$, with its probability density function $f(x)$ adhering to the truncated normal distribution, denoted as $N(4600,100^2)$, where the most likely demand $m=4600$. The solution yields that when $H'(w^*)=0$, the results are $w^*=576$, $\delta^*=(m-\beta w)/2\beta=172.0$, and the manufacturer's maximum expected revenue $M(w^*)=3.56\times10^5$.

Traditionally, many studies on CLSCs have relied on the aforementioned expected utility maximization as the decision criterion, assuming perfect rationality among decision-makers. However, with the ongoing evolution of behavioral economics and psychology, a series of experiments have increasingly highlighted the critical role of human behavioral preferences in shaping the members' decision. Decision-makers with diverse personality traits exhibit varying pricing or purchasing actions. For instance, in Schweitzer and Cachon's experiments (2000), retailers made purchase decisions based on given price and demand distributions, revealing significant deviations between the actual orders and the optimal order predictions assumed by economic agents. Therefore, the ensuing section delves into the behavioral preferences of the retailer as a "follower" in the CLSC, and analyzes the manufacturer's decision-making process comprehensively.

3. Closed loop supply chain model

Among various behavioral decision theories, we delve into the focus theory of choice. It is rooted in decision-makers' behavioral preferences for focal points or concerns when making decisions. The decision process described by the focus theory of choice entails a two-step strategy: initially, for each action, specific events are selected as the focal point by weighing the likelihood of the event occurring and the associated satisfaction. Subsequently, the decision-maker selects the most preferred action among all actions according to the focus principle. The focus theory of choice scrutinizes and elucidates individuals' decision processes under conditions of risk and uncertainty, which provides plausible rationalizations for the paradoxes in decision theory put forward by luminaries such as St. Petersburg, Allais, and Ellsberg and offers coherent explanations that address these enduring enigmas within the realm of decision-making. (Guo, 2019). It also addresses anomalies encountered in management and decision-making, including preference reversal, violation of random predominance, and transmissibility violation. Grounded in the concept of finite rationality proposed by Nobel laureate Simon, it offers the first axiomatization of the decision process's rationality, a proposition confirmed by psychological research (Simon, 1976; Stewart et al, 2016).

Decision-makers within the focus theory of choice are deemed to be finite rational, incapable of simultaneously focusing on all possible events, and instead weigh the benefits and probabilities jointly. Additionally, each individual has a different evaluation and focus on a particular event due to their different personality traits, so there are different evaluation systems for individuals in this theory, which are positive and negative evaluation systems. Under different systems, each individual focuses on relatively high probability events, but have different preferences for profits. When their preference is for substantial profits, the high-profit event is the positive focus. Conversely, the low-profit event is the negative focus. One of the evaluation systems is dominant for the decision-maker, while the other remains latent. The dominance of a particular evaluation system is contingent upon the decision-maker's personality characteristics. For instance, optimistic decision-makers prioritize a positive evaluation system, whereas pessimistic decision-makers typically emphasize a negative one. However, both evaluation systems may be activated simultaneously, leading to decision-makers experiencing indecision when making specific choices. Thus, the focus theory of choice provides a nuanced framework to describe decision-makers' behavioral patterns when confronted with risky decisions. Employing straightforward queries can assist in discerning which evaluation system holds positive significance for the decision-maker. This section examines the retailer's behavioral decision within a positive evaluation framework, treating the corresponding decision as a two-step process. In the initial step, the retailer assesses the positive demand focus for each possible market demand by comparing the revenue associated with all possible demands and their corresponding probabilities. In the subsequent step, the retailer determines the ideal profit pricing by scrutinizing the focus of all potential retail profits. Building upon the above discussion, we articulate the decision model for the retailer under the positive evaluation system.

To ensure that decision makers are able to make decisions based on their core concerns, the process of transforming the retailer's probability density function and benefit function into a relative likelihood function and a satisfaction function is critical for subsequent calculations and analysis. The details of this transformation process are presented in the subsequent detailed Definitions 1 and Definitions 2.

Definition 1 Let V be the domain of values of the retailer profit function $v_\delta(x, w, \delta)$. For any $v'_\delta, v''_\delta \in V$ such that $u(v'_\delta) > u(v''_\delta) \Leftrightarrow v'_\delta > v''_\delta$, and there exists $v^c_\delta \in V$ such that $u(v^c_\delta) = \max_{v\in V} u(v_\delta) = 1$, then $u: V \to [0,1]$ is termed the satisfaction function.

The composite satisfaction function of this paper, denoted as $u(x, w, \delta)$, is defined for any given wholesale price w and profit δ. It signifies the satisfaction level of the retailer when the realized value of potential demand in the market is x.

Given the premise assumption $x \in [l, h]$, the observed demand x enables the calculation of the potential maximum revenue for the retailer as $v^{max}_\delta = \frac{(h-w\beta)^2}{4\beta}$. When $\delta = 0$, the minimum retailer return is $v^{min}_\delta = 0$. Following Definition 1, the satisfaction function is expressed as

$$u(x, w, \delta) = \frac{u(x, w, \delta) - v^{min}_\delta}{v^{max}_\delta - v^{min}_\delta} = \frac{4\beta\delta(x - \beta(\delta + w))}{(h - w\beta)^2}. \tag{4}$$

Definition 2 For any $x_1, x_2 \in X$, it holds $\pi(x_1) > \pi(x_2) \Leftrightarrow f(x_1) > f(x_2)$, and there exists $x_c \in X$ such that $\pi(x_c) = \max_{x\in X} \pi(x) = 1$, then we call π: $X \to [0,1]$ is the relative likelihood function.

For any $\forall x \in X$, $\pi(x)$ is referred to as the relative likelihood. Given that the actual distribution closely approximates a normal distribution, the relative likelihood function for the normal distribution is employed as follows:

$\pi(x) = 1 - a\frac{(m-x)^2}{(h-m)^2}$,	

Here, $a \in (0,1]$, and m denotes the mean value of the demand. The function $\pi(x)$ is strictly increasing on $[l, m]$ and strictly decreasing on $[m, h]$. The relative likelihood function serves as a normalized probability density function, indicating the relative likelihood positions of different outcomes. The focus theory of choice selects de-quantified probability density and profit functions as the principal inputs for decision process(Edgeworth, 1888; Frank, 1985). Having transformed the profit function and probability density function into a satisfaction function and a relative likelihood function, the decision problem within the focus theory of choice is framed with satisfaction and likelihood as primary inputs. This decision framework envisions decision-making under risk as a two-step process: the first step identifies the most significant outcome (termed the focus) given all possible decision actions, and the second step involves selecting the best action by evaluating all relevant focuses. Subsequently, we proceed to construct and analyze the model within the focus theory of choice.

3.1 Retailer's decision model

Within the positive evaluation system, the retailer endeavors to pinpoint the demand characterized by both high satisfaction and likelihood. For every specified wholesale price w and retail profit δ, $X_p(w,\delta)$ is defined as the optimal solution set for the optimization problem:

$$\max_{x\in[l,h]} \{\varphi\pi(x)+u(x,w,\delta)\}, \tag{5}$$

Here, φ is a positive constant acting as a scaling factor, determining the weight assigned to $\pi(x)$. Given that $\pi(x)$ is a quadratic function and $u(x,w,\delta)$ is linear, their summation results in the Pareto optimal solution. The ensemble of all Pareto optimal solutions constitutes the Pareto optimal solution set. The objective function value associated with the Pareto optimal solution is termed the Pareto optimal frontier or Pareto frontier surface. The increase in φ tends to magnify $\varphi\pi(x)$, reducing the weight assigned to $u(x,w,\delta)$, thereby allowing for optimal x with an elevated likelihood. Conversely, the fall of φ diminishes $\varphi\pi(x)$, facilitating optimal x with a lower likelihood and relatively high satisfaction. Consequently, φ serves as a weight to balance retailer satisfaction and relative likelihood. A reduction in φ implies the retailer's willingness to sacrifice some probability to pursue higher returns, signifying the degree of retailer optimism. A lower φ value indicates greater retailer optimism. By employing (5) and Definition 2, it is evident that for any demand $x\in[l,h]$ and retail profit $\delta\in\Delta(w)=[0,l/\beta-w]$, there exists a unique solution in $X_p(w,\delta)$, referred to as the positive focus of δ and denoted as $x_p(w,\delta)$. Consequently, in the face of all potential positive foci, the optimal retail profit is derived through the following optimization problem:

$$\max_{\delta\in\Delta(\mathrm{w})} \left\{\kappa\pi\left(x_p(w,\delta)\right)+u\left(x_p(w,\delta),\ w,\delta\right)\right\}, \tag{6}$$

Here, κ is a positive constant, and the optimal solution and solution set of problem (6) are denoted by $\delta_p(w)$ and $\Delta_p(w)$, respectively. For $\delta_1,\delta_2\in\Delta_p(w)$, if $u\left(x_p(w,\delta_1),w,\delta_1\right)\geq u\left(x_p(w,\delta_2),w,\delta_2\right)$, and $\pi(x_p(w,\delta_1))\geq\pi(x_p(w,\delta_2))$, then we have $\max_{\delta\in\Delta(\mathrm{w})}\left\{\kappa\pi\left(x_p(w,\delta_1)\right)+u_r(x_p(w,\delta_1),w,\delta)\right\}\geq\max_{\delta\in\Delta(\mathrm{w})}\left\{\kappa\pi\left(x_p(w,\delta_2)\right)+u(x_p(w,\delta_2),w,\delta\right\}$. This implies that (6) can identify a retail profit with both relatively high likelihood and a high level of satisfaction. A higher κ enhances $\kappa\pi\left(x_p(w,\delta)\right)$ in contrast to $u\left(x_p(w,\delta),w,\delta\right)$, allowing for $\delta_p(w)$ with both higher satisfaction and lower likelihood. Conversely, a lower κ reduces $\kappa\pi(x_p(w,\delta))$ relative to $u\left(x_p(w,\delta),w,\delta\right)$, leading to lower likelihood and higher satisfaction for the existence of $\delta_p(w)$ as a positive focus. Since (6) identifies behaviors (profits) based on their positive focus, κ represents the retailer's confidence level in the decision. A diminished value of κ means more confidence of the retailer. Suppose that Δ_p contains a unique element in the system. In this case, Δ_p is termed the optimal retail profit within the positive system and is denoted by $\delta_p(w)$.

3.2 Analysis for the retailer's optimal decisions

This section is bifurcated into two segments: the retailer's positive focus and optimal retail profit under the focus theory of choice, based on the fundamental definitions, and the manufacturer's decision process for establishing the optimal wholesale price. For model (5), let

$$f(x)=\varphi\left(1-a\frac{(m-x)^2}{(h-m)^2}\right)+\frac{4\beta\delta\left(x-\beta(\delta+w)\right)}{(h-w\beta)^2}. \tag{7}$$

The derivative of $f(\cdot)$ is

$$f'(x)=\varphi\frac{2a(m-x)}{(h-m)^2}+\frac{4\beta\delta}{(h-w\beta)^2}.$$

The maximum point of $f(\cdot)$ is

$$x_\varphi=m+\frac{1}{\varphi}\cdot\frac{2\beta\delta(h-m)^2}{a(h-w\beta)^2}.$$

Lemma 1 The positive demand focus of retailer concerns is presented as follows:

$$x_p(w,\delta)=\min\{x_\varphi,h\}. \tag{8}$$

Moreover, it can be obtained that

(1) If $\varphi > \frac{2\beta\delta(h-m)}{a(h-w\beta)^2}$, i.e., $\delta < \frac{\varphi a(h-w\beta)^2}{2\beta(h-m)}$, $\delta \in [0, \delta_0]$, $x_\varphi < h$, then $x_p(w,\delta) = x_\varphi$.

(2) If $\varphi \le \frac{2\beta\delta(h-m)}{a(h-w\beta)^2}$, i.e., $\delta \ge \frac{\varphi a(h-w\beta)^2}{2\beta(h-m)}$, $\delta \in [\delta_0, \frac{l}{\beta} - w\ ]$, $x_\varphi \ge h$, then $x_p(w,\delta) = h$. Here, $\delta_0 = \min\left\{\frac{\varphi a(h-w\beta)^2}{2\beta(h-m)}, \frac{l}{\beta} - w\right\}$, $\delta_\varphi = \frac{\varphi a(h-w\beta)^2}{2\beta(h-m)}$, $\delta_h = \frac{l}{\beta} - w$.

See the appendix for the proof of Lemma 1.

For any given retail profit, Lemma 1 shows that φ is vital in determining the retailer's active demand focus: in the first case, φ reaches a higher range and the focal point is x_φ, which means that it has a diminished level of satisfaction at an increased likelihood; in the second case, φ takes a small enough value, meanwhile, the relative likelihood function at the focus $x_p(w,\delta) = h$; in the second case, the value is sufficiently small that the relative likelihood function has a significant effect at the focal point, and the focal point has a low likelihood at an elevated level of satisfaction. The range of positive focal points that retailers focus on under the positive evaluation system is $[m, h]$, with φ the value gradually decreasing, the $x_p(w,\delta)$ gradually from x_φ closer to h. According to Lemma 1, the following result can be derived.

Theorem 1 When $\delta \in [0, \delta_0]$, $x_p(w,\delta)$increases monotonically in δ, and $\pi\left(x_p(w,\delta)\right)$ is decreases monotonically in δ. When $\delta \in [\delta_0, \delta_m]$, $x_p(w,\delta)$ is constantly equal to h, the $\pi\left(x_p(w,\delta)\right) = h$ and $\pi\left(x_p(w,\delta)\right) = \pi(h) = 1 - a$.

See the appendix for the proof of Theorem 1.

Theorem 1 shows that the relationship between the positive focus $x_p(w,\delta)$ and the relative likelihood value of the positive focus $\pi\left(x_p(w,\delta)\right)$, and the retail profit δ. When the retail profit exceeds δ_0, the retailer prioritizes the highest potential demand in the market. As the value of δ shifts, for any $\delta_1, \delta_2 \in [0, \delta_0]$, if $\delta_1 < \delta_2$, then δ_1 of the positive focus is less than the δ_2 of the positive focus, signifying that an increase in retail profit leads retailers, under the positive evaluation framework, to favor higher demand. Meanwhile, $\pi\left(x_p(w,\delta)\right)$ on δ is continuous, as the retail profit incrementally rises, the relative likelihood value of the positive focus stays at the minimum value from the maximum value $\pi(m)$ decreases gradually to $\pi(h)$. As the retail profit increases, the relative likelihood value remains at the early minimum value $\pi(h)$. Drawing from the aforementioned Theorem and the Lemma, the optimal retail profit of retailer focus under the positive evaluation system is derived.

Theorem 2 The optimal concern of the retailer in the positive evaluation system $\delta_P(w)$ is expressed as follows:

(1) If $\varphi > \frac{2(l-\beta w)(h-m)}{a(h-\beta w)^2}$, then

$$\delta_P(w) = \begin{cases} \delta_h, & \kappa \le \kappa_1 \\ \delta_\kappa, & \kappa > \kappa_1 \end{cases}.$$

(2) If $\frac{h-m}{a(h-\beta w)} < \varphi \le \frac{2(l-\beta w)(h-m)}{a(h-\beta w)^2}$, then

$$\delta_P(w) = \begin{cases} \delta_\varphi, & \kappa \le \kappa_2 \\ \delta_\kappa, & \kappa > \kappa_2 \end{cases}.$$

(3) If $\varphi \le \frac{h-m}{a(h-\beta w)}$, then

$$\delta_P(w) = \begin{cases} \delta_c, & \kappa \le \kappa_3 \\ \delta_\kappa, & \kappa > \kappa_3 \end{cases}.$$

Here,

$$\delta_\kappa = \frac{\varphi^2 a(h-w\beta)^2(m-w\beta)}{2\beta(\kappa-2\varphi)(h-m)^2 + 2a\beta\varphi^2(h-w\beta)^2},\ \delta_c = \frac{h-\beta w}{2\beta},\ \delta_\varphi = \frac{\varphi a(h-w\beta)^2}{2\beta(h-m)},\ \delta_h = \frac{l}{\beta} - w,$$

$$\kappa_1 = 2\varphi + \frac{a\varphi^2(h-w\beta)^2(m+w\beta-2l)}{2(l-w\beta)(h-m)^2},\ \kappa_2 = 2\varphi + \frac{\varphi(m-w\beta)(h-m) - a\varphi^2(h-w\beta)^2}{(h-m)^2},$$

$$\kappa_3 = \frac{(2a\varphi+1)(h-m)^2 - a\varphi^2(m-w\beta)^2 + \sqrt{(a^2\varphi^2(h-w\beta)^2 - 2a\varphi(h-m)(h+w\beta-2m) + (h-m)^2)(a\varphi(h-w\beta)-h+m)^2}}{2a(h-m)^2}.$$

See the appendix for the proof of Theorem 2.

Optimal profit pricing is influenced by the retailer's confidence level φ and optimism κ. Retailers with different personalities influence the final decision-making results, which can be categorized as follows. A range of values for retail profit about the φ value and κ changes vary, as shown in Fig. 2.

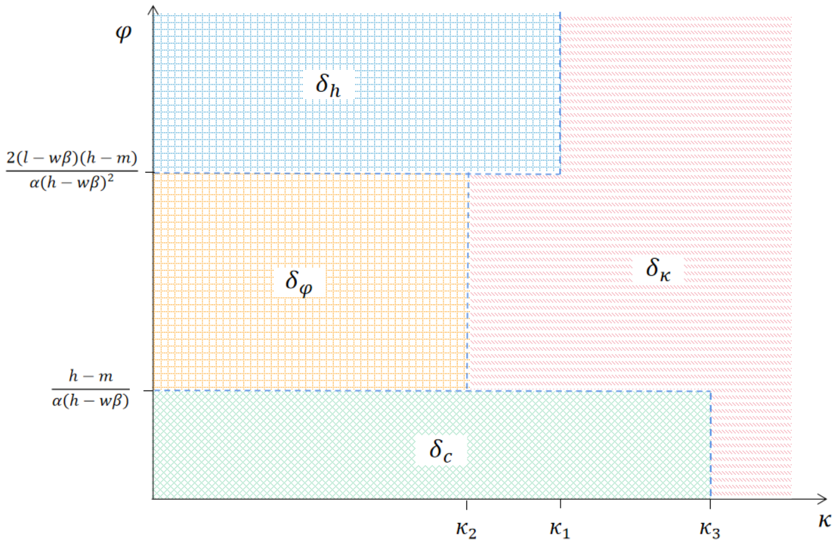

Fig. 2. The range of values of optimal retail profit

Case 1: When the retailer's personality type is low optimism, i.e., $\varphi > \frac{2(l-\beta w)(h-m)}{\alpha(h-\beta w)^2}$, its optimal profit pricing varies as the level of self-confidence changes.

(1) $\kappa > \kappa_1$ indicates that the retailer is less confident when the optimal profit pricing choice δ_κ.
(2) $\kappa \le \kappa_1$ indicates that the retailer is more confident when the optimal profit pricing choice δ_h.

Case 2: When the retailer's personality type is moderate optimism, i.e., $\frac{h-m}{\alpha(h-\beta w)} < \varphi \le \frac{2(l-\beta w)(h-m)}{\alpha(h-\beta w)^2}$, its optimal profit pricing varies with the level of self-confidence.
(1) $\kappa > \kappa_2$ indicates that the retailer is less confident when the optimal profit pricing choice δ_κ.
(2) $\kappa \le \kappa_2$ indicates that the retailer is more confident when the optimal profit pricing choice δ_φ.

Case 3: When the retailer's personality type is high optimism, i.e., $\varphi \le \frac{h-m}{\alpha(h-\beta w)}$, its optimal profit pricing varies with the level of self-confidence.
(1) $\kappa > \kappa_3$ indicates that the retailer is more confident when the optimal profit pricing choice δ_κ.
(2) $\kappa \le \kappa_3$ indicates that the retailer is more confident when the optimal profit pricing choice δ_c.

After considering the retailer's behavior to obtain its optimal retail profit, the manufacturer starts to make decisions. It is known that under the Stackelberg game, the manufacturer establishes its optimal wholesale price by first determining the retailer's personality characteristics and then predicting the retailer's optimal profit pricing w.

3.3 Manufacture's decision model

Since the decision model of the retailer is analyzed, now the expected return of the manufacturer is

$$H(v_m(X,w)) = \int_{n+\alpha b+\beta(\delta_p^*+w)}^{h} \begin{pmatrix} w\left(x-\beta(\delta_p^*+w)\right) - c_n\left(\left(x-\beta(\delta_p^*+w)\right)-(n+\alpha b)\right) - (c_o+b)(n+\alpha b) \\ +P\left(E - e_o(n+\alpha b) - e_n\left(\left(x-\beta(\delta_p^*+w)\right)-(n+\alpha b)\right)\right) \end{pmatrix} f(x)dx$$
$$+ \int_{l}^{n+\alpha b+\beta(\delta_p^*+w)} \begin{pmatrix} (w-c_o)\left(x-\beta(\delta_p^*+w)\right) + s\left((n+\alpha b)-\left(x-\beta(\delta_p^*+w)\right)\right) - b(n+\alpha b) \\ +P\left(E - e_o\left(x-\beta(\delta_p^*+w)\right)\right) \end{pmatrix} f(x)dx$$

The first-order derivative of the manufacturer's revenue with respect to w is

$$H'(w) = -\int_l^h F(x)dx + F\left(n+\alpha b+\beta(\delta_p^*+w)\right)(c_o+s+Pe_o-c_n-Pe_n)\beta(\delta_p^{*\prime}+1)$$
$$-(w-c_n-Pe_n) - \beta(\delta_p^*+w) + (-w+c_n+Pe_n)\beta(\delta_p^{*\prime}+1) + h.$$

The second-order derivative of the manufacturer's revenue with respect to w is

$$H''(w) = f\big(n + \alpha b + \beta(\delta_p^* + w)\big)\beta^2(\delta_p^{*\prime} + 1)^2(c_o + s + Pe_o - c_n - Pe_n) - 2\beta(\delta_p^{*\prime} + 1)$$
$$+F\big(n + \alpha b + \beta(\delta_p^* + w)\big)(c_o + s + Pe_o - c_n - Pe_n)\beta\delta_p^{*\prime\prime} + (-w + c_n + Pe_n)\beta\delta_p^{*\prime\prime} - 1$$

It can be seen that $H'(w)$ and $H''(w)$ can not be directly classified as positive or negative. We discuss by different situations, i.e., how the manufacturer determines the optimal wholesale price when confronted with retailers with different risk preferences.

Case 1. $H'(w) > 0$. In this case, $H(w)$ exhibits a monotonically increasing trend, which means that the manufacturer's profit rises with an increase in the wholesale price. In other words, when confronted with a positive retailer, the manufacturer can choose the maximum w within its acceptable range to optimize the profit, while the retailer follows the same principle in setting retail prices.

Case 2. $H'(w) < 0$. In this case, $H(w)$ exhibits a consistently decreasing trend, indicating that the manufacturer's profit diminishes as the wholesale price escalates. In other words, when confronted with a positive retailer, the manufacturer can choose the minimum w within its acceptable range to optimize the profit, while the retailer follows the same principle in setting retail prices.

Case 3. $H''(w) < 0$ and there exists w^* such that $H'(w) = 0$. Now $H(w)$ is strictly concave. When confronted with a positive retailer, the manufacturer can find an optimal w^* within its acceptable range to optimize the profit, while the retailer follows the same principle in setting retail prices.

Since the expectation function $H(w)$ of the upper-level manufacturer is continuous, there exists an optimal solution. Denote w^* as the optimal wholesale price. In the case that the manufacturer chooses w^*, the optimal profit of the retailer is $\delta_p(w^*)$, then the equilibrium solution of the CLSC is $\big(w^*, \delta_p(w^*)\big)$. Notice that the equilibrium solution of the CLSC is affected by the behavioral preferences of the lower-level retailers, i.e., φ and κ affect the equilibrium solution. Retailers with different personality traits own different $\delta_p(w^*)$ and different w^* can be obtained based on different $\delta_p(w^*)$. In particular, if $\delta_p(w^*) = \delta^*$, then this result is consistent with the result under the classical expectation. It can be seen that the conclusion under the classical expectation is only a case under the focal decision theory.

On the other hand, the equilibrium solution $\big(w^*, \delta_p(w^*)\big)$ and the profits of members are also affected by the carbon trading price P, carbon credits E, and the recycling price b. In order to further test these parameters' impact on the CLSC, we are going to conduct the sensitivity analysis through numerical experiments.

4. Numerical experiments and results analysis

This section delves into the dynamics of a manufacturer and a retailer operating within the framework of a carbon trading scheme. Specifically, Company A, situated in China, specializes in the production and recycling of electronic equipment, while Company B serves as a retailer of these electronic products. The operational structure involves Company A, the manufacturer, determining wholesale price w and supplying electronic products to Company B, the retailer. In turn, the retailer, Company B, establishes retail prices in response to market demand. Importantly, the recycling process conducted by Manufacturer A, denoted by a recycling price b, involves reclaiming waste products from consumers and manufacturing new products utilizing both raw and recycled materials. It is noteworthy that production costs associated with recycled materials are lower than those linked to raw materials. The retailer is entrusted with sales responsibilities and is exempt from inventory costs. Additionally, Manufacturer A's production activities fall under the jurisdiction of the carbon trading scheme, where the unit carbon emissions from recycled materials are notably lower than those arising from raw materials. Given the imperative of fostering a long-term and stable collaboration between them, the establishment of optimal wholesale and retail prices becomes a critical facet of their operational strategy.

4.1 Parameter setting

In this subsection, the involved parameters are set as follows: the manufacturer's unit production costs $c_n = 200$ and $c_o = 120$, unit recovery price $b = 30$, unit residual value $s = 20$, the consumer's sensitivity coefficient to the price $\beta = 5$, the consumer's sensitivity coefficient to the recovery price $\alpha = 20$, minimum recycling volume of the market $n = 100$, government-mandated carbon credit for the manufacturer in a single cycle $E = 4200$, carbon emissions of unit product $e_n = 7$ and $e_o = 5$, the price of unit carbon credits in the carbon trading market $P = 5$.

Assume that the market demand X is a random variable with an interval $[4200, 5000]$. The probability density function $f(x)$ obeys the truncated normal distribution, which is denoted as $N(4600, 100^2)$. The most likely demand $m = 4600$. It is known that the manufacturer is perfectly rational and selfish and decide its optimal wholesale price through an expectation-based approach. The retailer analyzes the optimal retail profits based on the focal decision theory under a positive evaluation system. Based on (4), the retailer's satisfaction function can be derived as

$$u(x,w,r)=\frac{20\delta(x-5(w+\delta))}{(5000-5w)^2}.$$

The relative likelihood function of the market potential demand function is

$$\pi(x)=1-\frac{(4600-x)^2}{160000}.$$

The probability density function of the market demand is

$$f(x)=\frac{1}{100\sqrt{2\pi}}\mathrm{e}^{-\frac{(x-4600)^2}{20000}}.$$

The distribution function is

$$F(x)=\frac{1}{100\sqrt{2\pi}}\int_l^x \mathrm{e}^{-\frac{(t-4600)^2}{20000}}\mathrm{d}t.$$

4.2 Focus preference

According to the parameter settings in the numerical experiments, the equilibrium solution $\{w^*,\delta_p(w^*)\}$ of the Stackelberg game can be solved. To be specific, the equilibrium solution of the game is the optimal price established by the manufacturer and the retailer respectively under the active evaluation system. Utilizing the inverse induction method, we initially need to determine the retailer's response function $\delta_p(w)$ and the corresponding optimal focus $\delta_p(w,\delta_p(w))$. According to Theorem 2, the optimal profit and the optimal positive focus of the retailer with different personality traits have different values. Therefore, it is imperative to determine the threshold values of φ and κ. Under different values, the optimal retail profit of the retailer is given based on Theorem 2. Specifically,

(1) If $\varphi>\frac{800(4200-5w)}{(5000-5w)^2}$, then

$$\delta_P(w)=\begin{cases}840-w, & \kappa\le\kappa_1,\\ \frac{\varphi^2(5000-5w)^2(4600-5w)}{1600000(\kappa-2\varphi)+10\varphi^2(5000-5w)^2}, & \kappa>\kappa_1.\end{cases}$$

(2) If $\frac{400}{5000-5w}<\varphi\le\frac{800(4200-5w)}{(5000-5w)^2}$, then

$$\delta_P(w)=\begin{cases}\frac{\varphi(5000-5w)^2}{4000}, & \kappa\le\kappa_2,\\ \frac{\varphi^2(5000-5w)^2(4600-5w)}{1600000(\kappa-2\varphi)+10\varphi^2(5000-5w)^2}, & \kappa>\kappa_2.\end{cases}$$

(3) If $\varphi\le\frac{400}{5000-5w}$, then

$$\delta_P(w)=\begin{cases}\frac{1000-w}{2}, & \kappa\le\kappa_3,\\ \frac{\varphi^2(5000-5w)^2(4600-5w)}{1600000(\kappa-2\varphi)+10\varphi^2(5000-5w)^2}, & \kappa>\kappa_3.\end{cases}$$

Here, $\kappa_1=2\varphi+\frac{\varphi^2(5000-5w)^2(5w-3800)}{320000(4200-5w)}$, $\kappa_2=2\varphi+\frac{400\varphi(14000-10w)-\varphi^2(5000-5w)^2}{160000}$, $\kappa_3=\frac{160000(2\varphi+1)-\varphi^2(4600-5w)^2+\sqrt{(\varphi^2(5000-5w)^2-800\varphi(5w-4200)+160000)(\varphi(5000-5w)-400)^2}}{320000}$.

In this context, the retailer's optimism φ and confidence κ are pivotal in ascertaining its optimal retail profit. By configuring these parameters, numerical results for the Stackelberg game can be obtained, elucidating the trend of outcomes across different values of φ and κ. According to the range of wholesale price, it can be seen that $w\in[200,680]$. In order to find the best wholesale price, by setting φ and κ, the optimal wholesale price w^*, the retail profit $\delta_p(w^*)$, the manufacturer's revenue $H(w^*)$, the satisfaction function $u\left(x_p(w^*,\delta_p(w^*)),w^*,\delta_p(w^*)\right)$ and the relative likelihood $\pi\left(x_p(w^*,\delta_p(w^*))\right)$ can be obtained. Therefore, we set $\varphi=0.1,\ 0.5,\ 1,\ 2,\ 10$ and $\kappa=0.1,\ 0.5,\ 1,\ 2,\ 10$.

Table 2
Equilibrium solutions ($\varphi=0.1$)

κ	w^*	$\delta_p(w^*)$	$H(w^*)$	$u\left(x_p,w^*,\delta_p(w^*)\right)$	$\pi\left(x_p\left(w^*,\delta_p(w^*)\right)\right)$
0.1	530	235.0	2.91×10^5	1.00	0
0.5	530	235.0	2.91×10^5	1.00	0
1	530	235.0	2.91×10^5	1.00	0
2	530	235.0	2.91×10^5	1.00	0
10	530	235.0	2.91×10^5	1.00	0

Table 3
Equilibrium solutions ($\varphi = 0.5$)

κ	w^*	$\delta_p(w^*)$	$H(w^*)$	$u\left(x_p, w^*, \delta_p(w^*)\right)$	$\pi\left(x_p\left(w^*, \delta_p(w^*)\right)\right)$
0.1	536	215.0	3.17×10^5	0.78	0.90
0.5	554	195.6	3.35×10^5	0.77	0.90
1	576	172.0	3.56×10^5	0.75	0.91
2	609	133.2	3.95×10^5	0.70	0.92
10	622	57.0	5.29×10^5	0.40	0.98

Table 4
Equilibrium solutions ($\varphi = 1$)

κ	w^*	$\delta_p(w^*)$	$H(w^*)$	$u\left(x_p, w^*, \delta_p(w^*)\right)$	$\pi\left(x_p\left(w^*, \delta_p(w^*)\right)\right)$
0.1	555	194.4	3.36×10^5	0.72	0.97
0.5	560	189.4	3.40×10^5	0.72	0.97
1	565	183.7	3.45×10^5	0.71	0.98
2	577	171.5	3.56×10^5	0.70	0.98
10	625	108.6	4.27×10^5	0.61	0.98

Table 5
Equilibrium solutions ($\varphi = 2$)

κ	w^*	$\delta_p(w^*)$	$H(w^*)$	$u\left(x_p, w^*, \delta_p(w^*)\right)$	$\pi\left(x_p\left(w^*, \delta_p(w^*)\right)\right)$
0.1	565	183.5	3.46×10^5	0.69	0.99
0.5	567	181.9	3.47×10^5	0.69	0.99
1	568	180.6	3.48×10^5	0.69	0.99
2	570	178.1	3.51×10^5	0.68	0.99
10	590	156.0	3.72×10^5	0.66	0.99

Table 6
Equilibrium solutions ($\varphi = 10$)

κ	w^*	$\delta_p(w^*)$	$H(w^*)$	$u\left(x_p, w^*, \delta_p(w^*)\right)$	$\pi\left(x_p\left(w^*, \delta_p(w^*)\right)\right)$
0.1	569	176.7	3.53×10^5	0.67	0.99
0.5	569	176.7	3.53×10^5	0.67	0.99
1	569	176.7	3.53×10^5	0.67	0.99
2	569	176.6	3.53×10^5	0.67	0.99
10	570	175.6	3.54×10^5	0.67	0.99

When the retailers' optimism level is high (i.e., $\varphi = 0.1$), as shown in Table 2, then regardless of κ, the retailer chooses the retail profit $\delta_p(w^*) = \delta_c = (h - \beta w)/2\beta$ as the optimal profit pricing. The manufacturer makes the highest profit with the wholesale price $w^* = 530$ and the retail profit $\delta_p(w^*) = 235.0$. In this case, the retailer exhibits considerable optimism regarding market demand. The demanding focus at this time is $x_p(w, \delta) = 5000$, the retail profit pricing is relatively high which is conform to retailers' positive focus criteria. if $\varphi = 0.1$, and $\kappa = 0.1$, it can be seen that $H(w)$ increases firstly and then decreases with w, the maximum value is reached at $w = 530$ and $H(w) = 2.91 \times 10^5$. When $\kappa = 0.5, 1, 2, 10$, the image is the same as in Fig. 3.

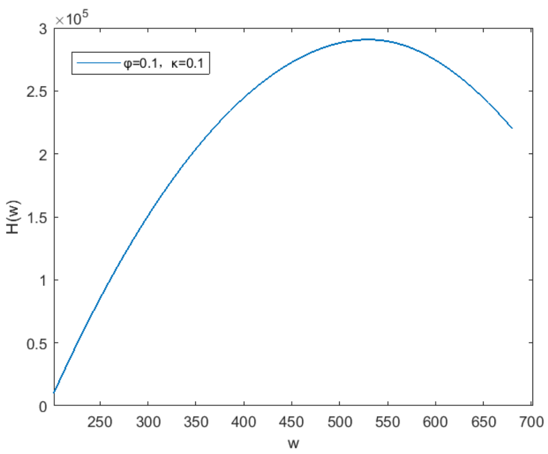

Fig. 3. Image of $H(w)$ (φ=0.1, κ= 0.1)

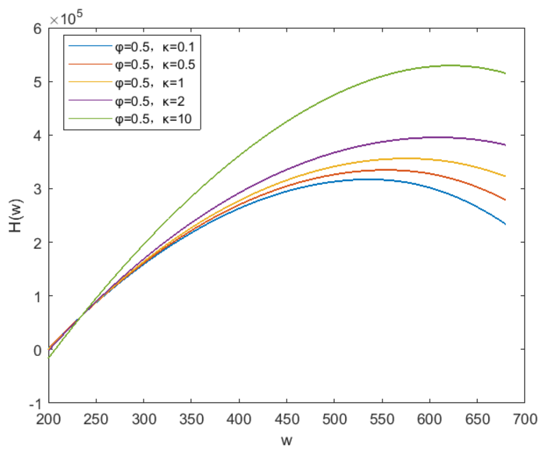

Fig. 4. Image of $H(w)$ (φ=0.5, κ=0.10.5,1,2,10)

As shown in Table 3, when the retailer's optimism is slightly reduced (i.e., $\varphi = 0.5$), the retailer's unit profit pricing choice

$\delta_p(w^*) = \delta_\kappa$. Fig. 4.2 illustrates the different pricing choices if $\varphi = 0.5$. Different values of κ will affect the manufacturer's achievement of optimal returns, it can be seen that when κ increases ($\kappa = 0.1 \to 10$), as the retailer's confidence level decreases, the manufacturer's optimal revenue increases. At this time, the manufacturer's revenue appears to fluctuate, when the manufacturer faces a retailer with a higher level of optimism($\varphi = 0.5$), the lower the level of retailer confidence, the higher the revenue for the manufacturer. When the retailer's optimism is moderate (i.e., $\varphi = 1$), as shown in Table 4, the retailer's profit pricing $\delta_p(w^*) = \delta_\kappa$. As the level of confidence decreases ($\kappa = 0.1 \to 10$), the retailer's retail profit pricing gradually decreases $\left(\delta_p(w^*) = 194.4 \to 108.6\right)$, the manufacturer's optimal wholesale price increases ($w^* = 555 \to 625$), the optimal profit available to the manufacturer also increases ($H(w) = 3.36 \times 10^5 \to 4.47 \times 10^5$). When retailers are less optimistic (i.e., $\varphi = 2, 10$), as shown in Tables 5 and 6, the trend of each indicator is the same as $\varphi = 1$. Figure 5 shows that when the φ is constant, as the κ increases, the axis of symmetry of the function image gradually moves to the right and gradually approaches the maximum $w = 680$. Since the unit retail profit pricing keeps decreasing, retailers' satisfaction decreases, and the relative likelihood keeps increasing.

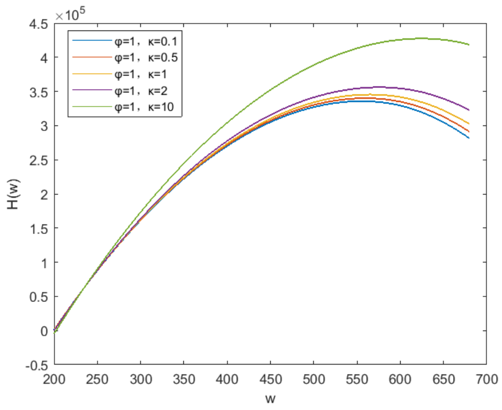

Fig. 5. Image of $H(w)$ (φ=1, κ=0.1,0.5,1,2,10)

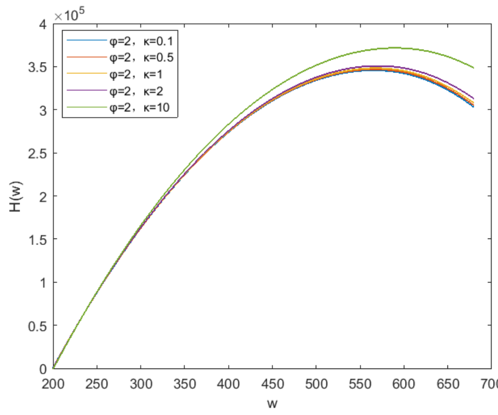

Fig. 6. Image of $H(w)$ (φ=2, κ=0.1,0.5,1,2,10)

Comparing Figs. 4-6 reveals that the impact of changes in κ becomes more pronounced as φ decreases. Notably, the influence of φ becomes more significant with greater value changes, thereby exerting a substantial impact on the outcome. The discernible trends in retailers' profit pricing, satisfaction level, and relative likelihood become more prominent as the confidence level escalates. The computational results presented in Tables 2-6 underscore the variability in retail profits among retailers with distinct levels of optimism and confidence within the positive evaluation system. Furthermore, these results unveil differences in key indicators such as the manufacturer's optimal wholesale price, retailer's optimal retail profit, manufacturer's expected profit, retailer's satisfaction, and relative likelihood across varying φ and κ. For highly optimistic retailers, regardless of their confidence level, the demand forecast for the market attains its peak, leading to the selection of a higher demand focus and elevated retail pricing. Confronted with such optimism, manufacturers opt for relatively lower wholesale prices, resulting in comparatively modest optimal revenue. In contrast, slightly optimistic retailers demonstrate that confidence levels significantly influence their decisions. As confidence decreases, a less assured market outlook prompts the adoption of lower pricing to stimulate demand. Consequently, the manufacturer's optimal revenue increases as profit pricing decreases. Collaborating with slightly optimistic yet less confident retailers is helpful to prove advantageous for promoting overall profitability.

In conclusion, retailers' optimism and confidence levels exert a positive influence on pricing decisions, aligning with real-world scenarios. These insights provide valuable understanding of retailers' behavioral choices in the supply chain and offer managerial guidance for retailers with diverse personalities.

4.3 Carbon trading prices and carbon credits

Manufacturers operating within the carbon trading framework have the option to participate in carbon credit transactions with the market. At the conclusion of each cycle, a manufacturer with total carbon emissions below the government-mandated limit can sell surplus credits to the carbon trading market, while those exceeding the limit necessitate the purchase of additional credits. Hence, the CLSC is influenced by the carbon trading price P and the government-imposed single-cycle carbon credits E. The dynamics of these factors can exert diverse effects on supply chain decisions.

Based on parameters setting in Section 4.1, a medium level of optimism and confidence is selected($\varphi = 1，\kappa = 1$). The numerical experiment is conducted for retailers with medium optimism and confidence level to analyze the carbon trading price given by the carbon trading market P. Table 7 illustrates the impact of carbon trading prices from the carbon market on retailers' profit pricing, manufacturers' wholesale prices, and manufacturers' income.

Table 7
Manufacture's profit on carbon trading price

P	w	δ	$H(w)$	Total carbon emissions e
0	549	191.5	3.48×10^5	5303.4
5	565	183.7	3.45×10^5	5029.9
10	580	176.4	3.44×10^5	4774.0
20	605	152.6	3.45×10^5	4348.3
50	659	138.1	3.66×10^5	3433.5

As shown in the table, when the carbon trading price is 0, the manufacturer's production activity is not under the constraint of the carbon trading scheme. It can emit carbon freely as a comparison reference. The following information can be obtained from the experimental results in the table. Carbon trading price P has an impact on the manufacturer's expected returns. With the increase of the value of P (from 0 to 10), the manufacturer's expected return starts to decrease because the manufacturer's carbon emissions are larger than the government's carbon emission allowance ($E = 4200$). Therefore, the manufacturer acquires some carbon credits from the carbon trading market. The benefits to the manufacturer are negatively correlated, consistent with the fundamental speculation of this work. However, as the value of P further increase (from 10 to 50), the benefits to manufacturers begin to increase because as the value of P rises, the manufacturer and retailer raise prices, which results in lower demand in the market. The number of products produced by manufacturing starts to decrease. Carbon emissions also decrease, and although some profits from selling products are lost, carbon emissions are also lower than the number of credits in a single cycle. At this time, P is positively correlated with the manufacturer.

Thus, when the carbon trading price P is given, manufacturers can decide their wholesale prices based on the high or low P. When the value of P is elevated, the manufacturer will increase the price to some extent to influence the market demand and thus reduce the internal production and carbon emission to save carbon emission costs. When the price is too high, the manufacturer has the opportunity to generate revenue by selling surplus carbon credits in the market. It is achieved through controlling the company's carbon emissions below the regulatory cap imposed by the government.

From Table 7, it is apparent that higher carbon trading prices correspond to lower achievable carbon emissions. However, the carbon trading market does not indiscriminately escalate prices since excessively high prices may prompt enterprises to continuously curtail production, focusing solely on selling carbon allowances in the trading market. In extreme cases, enterprises might cease production entirely, engaging exclusively in the sale of carbon emission rights. To prevent disruptions to the product market's supply and demand equilibrium, the carbon trading market typically imposes limits on trading prices, maintaining them within a reasonable range.

The carbon emission amount assumed in the above experiment is $E = 4200$. It can be seen from the results of the above experiment that when the manufacturer's carbon emissions are higher than 4200, the manufacturer's gain is less than $P = 0$. When the carbon emission is less than 4200, the manufacturer's benefit is more than $P = 0$. When the carbon emission is less than 4200, the manufacturer's benefit is more significant from the government. It can be seen that the manufacturer's gain has a special relationship with the carbon emission allowance set by the government, and next, keeping $P = 5$ unchanged and analyzing changes.

Table 8
Manufacturer's profit on carbon trading price

Carbon credits	w	δ	$H(w)$	Total carbon emissions
2200	565	183.7	3.35×10^5	5029.9
3200	565	183.7	3.40×10^5	5029.9
4200	565	183.7	3.45×10^5	5029.9
5200	565	183.7	3.50×10^5	5029.9
6200	565	183.7	3.55×10^5	5029.9

As shown in Table 10, wholesale prices, retail pricing, and total carbon emissions do not change when carbon credits are changed. However, manufacturers' revenues and carbon credits are positively correlated, and $\frac{\partial H(w)}{\partial E} = P = 5$. When the credits exceed the total amount of carbon emissions, $e = 5029.9$. When the credits exceed the total carbon emissions, the manufacturer can sell the remaining carbon credits to gain revenue, and the manufacturer is the beneficiary of the carbon trading policy. The wholesale price is not affected by carbon credits, and this result is consistent with the previous theoretical analysis. Within the scheme of the carbon trading, manufacturers, as producers, must balance sales profits and carbon emission costs. External carbon trading prices directly influence manufacturers' pricing decisions. Lower carbon trading prices prompt manufacturers to adopt lower wholesale prices, consequently leading to reduced retail prices, expanded market demand, and increased production. Conversely, higher carbon trading prices drive manufacturers to elevate wholesale prices and reduce production to mitigate carbon trading costs. While the implementation of a carbon trading scheme by government entities reduces enterprise carbon emissions, excessively high carbon trading prices may induce some enterprises to forgo production, opting to sell carbon emission rights directly—an outcome detrimental to effective market operations.

4.4 Recovery prices

The recycling price is crucial in the dynamics of a CLSC, directly affecting the recycling volume. Therefore, it is imperative to scrutinize the sensitivity of the recycling price to the manufacturer's revenue function. In this study, we focus on the manufacturer's recycling model exclusively. While this paper delves into a specific recycling model, it is essential to acknowledge the existence of various recycling models in society. These models may involve the retailer taking responsibility for recycling, third-party recycling companies handling the process, or multiple entities engaging in recycling simultaneously. Based on the parameters setting in Section 4.1, the optimism level and confidence level were selected as high($\varphi = 0.1$，$\kappa = 0.1$) optimism and confidence level, medium optimism and confidence level ($\varphi = 1$，$\kappa = 1$) and low levels of optimism and confidence ($\varphi = 10$，$\kappa = 10$) retailers to perform numerical experiments to evaluate how the price b affect the retailer's profit pricing, the manufacturer's wholesale price, and corresponding revenue.

Table 9
Changes of manufacturer's profit ($\varphi = 0.1$, $\kappa = 0.1$)

b	w	δ	$H(w)$
10	538	231.0	2.74×10^5
20	537	231.5	2.85×10^5
30	530	235.0	2.91×10^5
40	510	245.0	2.88×10^5
50	503	248.5	2.73×10^5

Table 10
Changes of manufacturer's profit ($\varphi = 1$, $\kappa = 1$)

b	w	δ	$H(w)$
10	567	182.7	3.28×10^5
20	567	182.7	3.38×10^5
30	565	183.7	3.45×10^5
40	549	191.0	3.44×10^5
50	534	198.8	3.33×10^5

Table 11
Changes of manufacturer's profit ($\varphi = 10$, $\kappa = 10$)

b	w	δ	$H(w)$
10	577	177.9	3.37×10^5
20	576	178.3	3.48×10^5
30	575	178.8	3.55×10^5
40	559	186.6	3.54×10^5
50	543	194.5	3.43×10^5

As shown in Tables 9-11, the recovery price b choice affects the wholesale price, retail pricing, and the manufacturer's expected revenue. When the recycling price b rises, the wholesale price gradually decreases, the retail profit gradually rises, and the manufacturer's expected revenue initially climbs before subsequently falling at $b = 30$. The logic behind this is intuitive. When the recycling price is low, the manufacturer recycles less and saves less remanufacturing and carbon reduction costs, so the profit decreases. When the recycling price is high, although the number of recycling increases, the manufacturer needs to pay a higher cost for recycling, and the revenue will be lower at this time. Analysis of the table reveals that as the recycling price b rises, the manufacturer sets a decreasing wholesale price. This phenomenon can be attributed to the excess recycling surpassing market demand, causing production costs and carbon emission reduction costs to decrease. Consequently, the manufacturer adjusts pricing downward to reflect reduced costs. Retailers respond to the rising wholesale price by gradually increasing retail profit pricing, as they believe higher recycling prices can stimulate sales and have confidence in market demand.

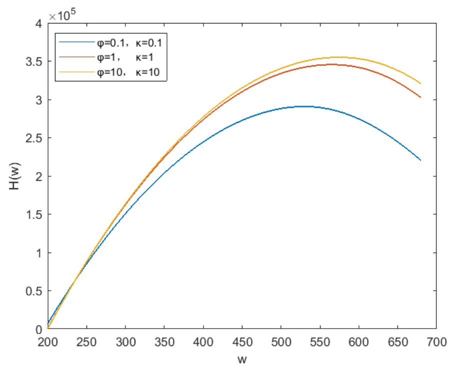

Fig. 7. Benefit of the manufacturer

For retailers with varying personality traits, the manufacturer achieves optimal returns when $b = 30$. However, discrepancies in optimal returns become apparent. Illustrated in Figure 6, manufacturers selecting less optimistic and less confident retailers ($\varphi = 10$ and $\kappa = 10$) yield higher returns than those partnering with highly optimistic and confident retailers ($\varphi = 0.1, 1$, and $\kappa = 0.1, 1$). This discrepancy underscores that manufacturers can benefit from choosing less optimistic and less confident retailers as partners.

5. Conclusions

Within the framework of recycling and carbon trading policies in a CLSC, this paper presents a novel manufacturer-led recycling model, incorporating the focus theory of choice to explore behavioral preferences of retailers as followers. Additionally, it delves into the decision biases of decision-makers within the positive evaluation system, specifically addressing uncertainties in demand. Our proposed model rejects the assumption of complete rationality and instead adopts the theory of focal choice, which emphasizes that decision makers, under the constraints of limited rationality and attentional resources, are more inclined to focus on the events or situations that they perceive to be the most critical. The introduction of this theoretical perspective provides us with a new dimension for understanding and modeling the Stackelberg game in the secondary supply chain composed of manufacturers and retailers. Within this framework, we consider the strategic interactions among supply chain members through a combination of theoretical and numerical approaches, and also focus on analyzing the differences in the decision-making process of suppliers with different personality traits, as well as the effect of optimistic degrees and confidence levels on the final decision outcomes.

This study contributes significantly to existing literature in several ways. Firstly, it introduces a new CLSC model, intertwining behavioral analysis of retailers within the manufacturer-dominated CLSC using the focus theory of choice. Secondly, the numerical analysis unequivocally demonstrates the impact of retailers' optimism and confidence levels on optimal retail profits. Findings reveal that decreased optimism and confidence prompt retailers to opt for lower retail pricing, thereby bolstering market demand and increasing the likelihood of event occurrence. Contrarily, retailers exhibiting greater optimism and self-confidence focus on the retail margin aligned with the most probable market demand. Manufacturers exhibit a preference for partnering with retailers who are relatively optimistic and less confident, with such retailers exerting a catalytic influence on manufacturers to garner more revenue. Furthermore, we delve into the influence of carbon trading prices on manufacturers' revenue in the carbon trading market. Manufacturers can tailor wholesale prices in response to given carbon trading prices. Specifically, wholesale prices rise as carbon trading prices rise. This, in turn, affects market demand, reducing internal production and carbon emissions, thereby cutting carbon emission costs. In instances of excessively high carbon trading prices, manufacturers can maximize revenue by selling surplus carbon credits in the market while maintaining carbon emissions below the government's limit. The recycling price is identified as a pivotal factor impacting wholesale prices, retail pricing, and the manufacturer's expected revenue. When the recycling price rises, the wholesale price decreases, retail profit increases gradually, and the manufacturer's expected return initially ascends before descending. Low recycling prices result in decreased recycling, leading to lower remanufacturing and carbon reduction costs and reduced profits. Conversely, high recycling prices, while increasing recycling volumes, incur higher recycling costs, leading to lower revenue.

It is noteworthy that this research assumes the manufacturer is accountable for recycling waste items and setting recycling prices in the CLSC. The presented recycling model is just one among many, and other recycling models exist, including third-party recycling, retailer recycling, and co-recycling. The exploration of the application of the focus theory of choice within the context of CLSCs warrants further investigation and scrutiny. Additionally, this study highlights the potential for integrating other behavioral research theories into the supply chain in the future.

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Appendix

1. Proof of Lemma 1: Considering the quadratic calculation within the framework of the normal distribution, the optimization problem (7) is formulated to determine the optimal solution for $f(x)$. It is not difficult to see that $x_\varphi \geq m$, then (8) holds.

2. Proof of Theorem 1: If $\delta \in [0, \delta_0]$, then $x_p(w,\delta) = x_\varphi$ and $x_p(w,\delta)$ increases in δ; if $\delta \in [\delta_0, \delta_m]$, then $x_p(w,\delta) = h$. Note that $\pi(\cdot)$ increases in $[l, m]$ and decreases in $[m, h]$. Besides, the function $\pi(\cdot)$ is monotonically decreasing. Therefore, the composition function $\pi\left(x_p(w,\delta)\right)$ is monotonically decreasing on $[w, \delta_0]$ with respect to δ and $\pi\left(x_p(w,\delta)\right) = 1 - a$ for any $\delta \in [\delta_0, \delta_m]$.

3. Proof of Theorem 2: For the optimization problem (7), let

$$g(\delta) = \kappa\pi\left(x_p(w,\delta)\right) + u\left(x_p(w,\delta), w, \delta\right).$$

Due to the expression of $x_p(w,\delta)$, we further get

$$g(\delta) = \kappa\left(1 - a\frac{(m-x_p(w,\delta))^2}{(h-m)^2}\right) + \frac{4\beta\delta(x_p(w,\delta)-\beta(\delta+w))}{(h-w\beta)^2}.$$

According to Lemma 1, it can be categorized into two distinct cases

Case 1: If $\delta \in [0, \delta_0]$, then $x_p(w,\delta) = x_\varphi$ and

$$g_1(\delta) = \kappa\left(1 - a\frac{(-\frac{2\beta\delta(h-m)^2}{\varphi a(h-w\beta)^2})^2}{(h-m)^2}\right) + \frac{4\beta\delta\left(m+\frac{2\beta\delta(h-m)^2}{\varphi a(h-w\beta)^2}-\beta(\delta+w)\right)}{(h-w\beta)^2}.$$

It is evident that the function $g_1(\cdot)$ is quadratic with respect to δ, the symmetry axis $\delta_\kappa = \frac{\varphi^2 a(h-w\beta)^2(m-w\beta)}{2\beta(\kappa-2\varphi)(h-m)^2+2a\beta\varphi^2(h-w\beta)^2}$, the quadratic coefficient $A = \frac{4\beta^2}{(h-w\beta)^2}\left(\frac{(2\varphi-\kappa)(h-m)^2}{\varphi^2 a(h-w\beta)^2} - 1\right)$.

① If $\kappa < 2\,\varphi - \frac{\varphi^2 a(h-\beta w)^2}{(h-m)^2}$, then $A > 0$ and $\delta_\kappa < 0$, which means that $\delta_1^* = \delta_0$.

② If $\kappa \geq 2\,\varphi - \frac{\varphi^2 a(h-\beta w)^2}{(h-m)^2}$, then $A \leq 0$ and $\delta_\kappa \geq 0$. The symmetry axis on $[0, \delta_0]$ can be classified and the optimal retail profit is

$$\delta_1^* = \begin{cases} \delta_0, & \delta_\kappa > \delta_0, \\ \delta_\kappa, & 0 \leq \delta_\kappa \leq \delta_0\,. \end{cases}$$

Case 2: If $\delta \in [\delta_0, \delta_h]$, then $x_p(w,\delta) = h$ and

$$g_2(\delta) = \kappa(1-a) + \frac{4\beta\delta(h-\beta(w+\delta))}{(h-\beta w)^2}.$$

It is not difficult to see that the function $g_2(\cdot)$ is quadratic with respect to δ and the symmetry axis $\delta_c = \frac{h-\beta w}{2\beta}$. Similarly, the optimal retail profit is

$$\delta_2^* = \begin{cases} \delta_c, & \delta_0 \leq \delta_c \leq \delta_h, \\ \delta_0, & \delta_h \leq \delta_0. \end{cases}$$

By comparing δ_1^* and δ_2^*, we get

(1) If $\kappa < 2\,\varphi - \frac{\varphi^2 a(h-\beta w)^2}{(h-m)^2}$, then

$$\delta_1^* = \delta_0,\ \ \delta_2^* = \begin{cases} \delta_c, & \delta_0 \leq \delta_c \leq \delta_h, \\ \delta_0, & \delta_c \leq \delta_0. \end{cases}$$

① If $\varphi < \frac{2(l-\beta w)(h-m)}{a(h-\beta w)^2}$, then $\delta_\varphi \leq \delta_h$ and $\delta_0 = \delta_\varphi$. It follows from $\varphi < \frac{h-m}{a(h-\beta w)}$ that $\delta_0 \leq \delta_c$. Let $\delta_2^* = \delta_c$ and $\varphi > \frac{h-m}{a(h-\beta w)}$, which implies that $\delta_0 > \delta_c$. By taking $\delta_2^* = \delta_0$, it yields that

$$\delta_2^* = \begin{cases} \delta_c, & \varphi < \frac{h-m}{a(h-\beta w)}, \\ \delta_0, & \frac{h-m}{a(h-\beta w)} < \varphi < \frac{2(l-\beta w)(h-m)}{a(h-\beta w)^2}. \end{cases}$$

The optimal solution of retail profit is

$$\delta^* = \begin{cases} \delta_c, & \varphi < \frac{(h-m)}{a(h-w\beta)}, \\ \delta_0, & \frac{(h-m)}{a(h-w\beta)} < \varphi < \frac{2(l-w\beta)(h-m)}{a(h-w\beta)^2}. \end{cases}$$

② If $\varphi > \frac{2(l-w\beta)(h-m)}{a(h-w\beta)^2}$, then $\delta_\varphi > \delta_h$ and $\delta_0 = \delta_h$. The optimal solution of retail profit is $\delta^* = \delta_h$. Assume that $2(l-w\beta) > h - w\beta$, it follows that $w \leq \frac{2l-h}{\beta}$.

The optimal set of solutions can be obtained if $\kappa < 2\,\varphi - \frac{\varphi^2 a(h-w\beta)^2}{(h-m)^2}$. The optimal set of solutions for retail profit is

$$\delta^* = \begin{cases} \delta_c, & 0 < \varphi \le \frac{h-m}{a(h-w\beta)}, \\ \delta_\varphi, & \frac{h-m}{a(h-w\beta)} < \varphi \le \frac{2(l-w\beta)(h-m)}{a(h-w\beta)^2}, \\ \delta_h, & \varphi > \frac{2(l-w\beta)(h-m)}{a(h-w\beta)^2}. \end{cases}$$

(2) If $\kappa \ge 2\varphi - \frac{\varphi^2 a(h-w\beta)^2}{(h-m)^2}$, then

$$\delta_1^* = \begin{cases} \delta_0, & \delta_\kappa > \delta_0, \\ \delta_\kappa, & 0 \le \delta_\kappa \le \delta_0, \end{cases} \quad \delta_2^* = \begin{cases} \delta_c, & \delta_0 \le \delta_c \le \delta_h, \\ \delta_0, & \delta_h \le \delta_0. \end{cases}$$

① If $\varphi < \frac{2(l-w\beta)(h-m)}{a(h-w\beta)^2}$, then $\delta_\varphi \le \delta_h$ and $\delta_0 = \delta_\varphi$.

(i) If $\delta_\kappa < \delta_\varphi$ and $\delta_c < \delta_\varphi$, it follows that $\kappa > 2\varphi + \frac{\varphi(m-w\beta)(h-m)-a\varphi^2(h-w\beta)^2}{(h-m)^2}$ and $\frac{h-m}{a(h-w\beta)} < \varphi < \frac{2(l-w\beta)(h-m)}{a(h-w\beta)^2}$, which implies that $g_1(\delta_1^*) = g_1(\delta_\kappa) > g_2(\delta_0) = g_2(\delta_2^*)$ and $\delta^* = \delta_\kappa$.

(ii) If $\delta_\kappa > \delta_\varphi$ and $\delta_c > \delta_\varphi$, it follows that $\kappa < 2\varphi + \frac{\varphi(m-w\beta)(h-m)-a\varphi^2(h-w\beta)^2}{(h-m)^2}$, $\varphi < \frac{h-m}{a(h-w\beta)}$, which means that $g_1(\delta_1^*) = g_1(\delta_0) < g_2(\delta_h) = g_2(\delta_2^*)$ and $\delta^* = \delta_h$.

(iii) If $\delta_\kappa > \delta_\varphi$ and $\delta_c < \delta_\varphi$, it follows that $\kappa < 2\varphi + \frac{\varphi(m-w\beta)(h-m)-a\varphi^2(h-w\beta)^2}{(h-m)^2}$ and $\frac{h-m}{a(h-w\beta)} < \varphi < \frac{2(l-w\beta)(h-m)}{a(h-w\beta)^2}$, which implies that $g_1(\delta_1^*) = g_1(\delta_\varphi) = g_2(\delta_\varphi) = g_2(\delta_2^*)$ and $\delta^* = \delta_\varphi$.

(iv) If $\delta_\kappa < \delta_\varphi$ and $\delta_c > \delta_\varphi$, it follows that $\kappa > 2\varphi + \frac{\varphi(m-w\beta)(h-m)-a\varphi^2(h-w\beta)^2}{(h-m)^2}$, $\varphi < \frac{h-m}{a(h-w\beta)}$. Furthermore, we have

$g_1(\delta_1^*) = g_1(\delta_\kappa) = \frac{a\varphi^2(m-w\beta)^2}{(h-m)^2(\kappa-2\varphi)+a\varphi^2(h-w\beta)^2} + \kappa$, $g_2(\delta_2^*) = g_2(\delta_c) = \kappa(1-a)+1$,

$g_1(\delta_\kappa) - g_2(\delta_c) = \frac{a\varphi^2(m-w\beta)^2}{(h-m)^2(\kappa-2\varphi)+a\varphi^2(h-w\beta)^2} + a\kappa - 1.$

Let $g_1(\delta_\kappa) - g_2(\delta_c) = 0$, it follows that

$$\kappa_3 = \frac{(2a\varphi+1)(h-m)^2 - a\varphi^2(m-w\beta)^2 + \sqrt{(a^2\varphi^2(h-w\beta)^2 - 2a\varphi(h-m)(h+w\beta-2m) + (h-m)^2)(a\varphi(h-w\beta)-h+m)^2}}{2a(h-m)^2},$$

$$\kappa_4 = \frac{(2a\varphi+1)(h-m)^2 - a\varphi^2(m-w\beta)^2 - \sqrt{(a^2\varphi^2(h-w\beta)^2 - 2a\varphi(h-m)(h+w\beta-2m) + (h-m)^2)(a\varphi(h-w\beta)-h+m)^2}}{2a(h-m)^2}.$$

It can be seen that $\kappa_2 < 2\varphi + \frac{\varphi(m-w\beta)(h-m)-a\varphi^2(h-w\beta)^2}{(h-m)^2} < \kappa_1$. Due to $\varphi < \frac{h-m}{a(h-w\beta)}$ and $2\varphi + \frac{\varphi(m-w\beta)(h-m)-a\varphi^2(h-w\beta)^2}{(h-m)^2} < \kappa < \kappa_1$, we have $\delta^* = \delta_c$. Due to $\varphi < \frac{h-m}{a(h-w\beta)}$ and $2\varphi + \frac{\varphi(m-w\beta)(h-m)-a\varphi^2(h-w\beta)^2}{(h-m)^2} < \kappa < \kappa_1$, we have $\delta^* = \delta_\kappa$.

By adding above four cases, the optimal retail profit is

$$\delta^* = \begin{cases} \delta_\kappa, & \kappa \ge \kappa_2,\ \varphi > \dfrac{h-m}{a(h-w\beta)}, \\ \delta_\varphi, & \kappa < \kappa_2,\ \varphi > \dfrac{h-m}{a(h-w\beta)}, \\ \delta_c, & \kappa < \kappa_3,\ \varphi < \dfrac{h-m}{a(h-w\beta)}, \\ \delta_\kappa, & \kappa \ge \kappa_3,\ \varphi < \dfrac{h-m}{a(h-w\beta)}. \end{cases}$$

② If $\varphi \ge \frac{2(l-w\beta)(h-m)}{a(h-w\beta)^2}$, then $\delta_\varphi > \delta_h$ and $\delta_0 = \delta_h$.

Therefore, it can be obtained that

$$\delta^* = \begin{cases} \delta_m, & \kappa < \kappa_1, \\ \delta_\kappa, & \kappa \ge \kappa_1. \end{cases}$$

Based on the above analysis, Theorem 2 is proved.