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Determining a manufacturing-delivery policy for a multi-item EPQ system with multi-shipment, quality assurance, overtime, postponement, and external source

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A B S T R A C T

Facing current client expectations for high quality, timely order response, and multiple shipments of various needed merchandise, today's producers must simultaneously satisfy external requirements and operate internally with minimum overall expenses and capacity constrained. Aiming to help present-day producers achieve the operational goals mentioned above, this work develops a decisional scheme to determine the best manufacturing-delivery policy for a multi-item economic production quantity (EPQ) system with multi-shipment, quality assurance, overtime, postponement, and external source. Combining a production postponement strategy in our multi-item batch fabricating procedures intends to first make all required standard/common parts for various client-needed merchandise and make finished goods in the 2nd phase. Two fabricating-uptime-shortening strategies are adopted: contracting out a proportion of the standard part's batch and overtime-making of finished goods. We include screening and rework tasks in fabricating procedures to help us remove the identified scraps and correct the repairable faulty items. The quality-assured finished batches are divided into multiple equal-amount shipments transported to meet client requests. The overall manufacturing-transportation relevant expenses, including quality and uptime-expedited costs, are mathematically modeled and minimized using optimization methodology to help derive the best manufacturing-delivery operating policy. Moreover, we offer an illustration to validate the results and our research scheme's capability numerically. This work mainly contributes to the literature by presenting a practical decision-making model. It enables the producers to expose numerous crucial problem-related managerial insights to facilitate producers in deciding the most appropriate manufacturing-delivery policy to meet clients' multi-criteria demands.

1. Introduction

This research will determine a manufacturing-delivery policy for a multi-item EPQ system with multi-shipment, quality assurance, postponement, external source, and overtime. When scheduling the fabrication of multiple products with commonality on a single piece of equipment, the management always examines the potential cost-savings/uptime-reduction benefits of applying a postponement strategy. Mendonça and Dias (2007) examined the effect of the postponement of final automobiles on Portugal's auto market logistical systems. Their study found that Portugal's automobile transporter secured the absolute quality of automobile manufacturers' products and delivered them to customers. The postponement consideration of the end product's finalized operations includes final testing and the possible breakdown and damage repairs in the end-product transporting process. Hence, the postponement of finalized operations affects the end-product reliability and the brand's image as important as that in the manufacturer's output ends. Pero et al. (2015) aimed to incorporate product modularity into the shipbuilding and construction industries' engineer-to-order (ETO) supply chain (SC). Using the

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explorative case studies approach, the researchers looked into the ETO industry's modularity concept. The study clarified the specific modularity meaning in the ETO industry. It examined the relationship between SC integration and modularity in terms of some significant/identifiable variables, e.g., IP awareness, innovativeness, company and product size, and customization. As a result, the relations found served as the foundation for future research in the studied areas. Kim et al. (2022) aimed to improve client responsiveness and reduce the stock level of finished merchandise by proposing inventory models to industrial practitioners with the "order-up-to" inventory policy. Their work was motivated by a leading Korean semiconductor company that intended to develop an inventory-management approach for a die bank with a fabricating wafers facility. The researchers used Arena for their simulation experiments. Their work generated sufficient evidence through pilot tests to justify the need for introducing a die bank. The results showed that by delaying the differentiation point of finished merchandise fabrication, a die bank created a more responsive supply chain facing demand changes. Other works (Pourakbar & Dekker, 2012; Galizia et al., 2020; Chiu et al., 2021; Malladi et al., 2021; Ramón-Lumbierres et al., 2021; Al-Hakimi et al., 2022; Kiani et al., 2022; Yang & Zhang, 2022; Manthirikul et al., 2023; Pushpalatha et al., 2024) studied the impact of different strategies of postponement/ delayed-differentiation on the planning and management of manufacturing firms and supply chains. To further shorten the multiproduct manufacturing uptime with the postponement, using an external source to expedite common parts' fabrication and implementing overtime to reduce end merchandise making are considered in our work to enhance the producer's competitive advantage. Anger (2008) presented an empirical signaling model to explain how unpaid overtime can benefit the industry. The researcher used 1993–2004's German socio-economic panel study data to investigate the relationship between overtime and future benefit. As a result, the study confirmed unpaid overtime's positive signaling value in West German workers. Ameknassi et al. (2016) incorporated logistics outsourcing related-factors such as security of supplies, customer segmentation, and extended producer responsibility into a stochastic multiproduct, multi-period, multi-objective supply chain model. Their study aimed to simultaneously minimize the anticipated supply chain logistics expenses and greenhouse gas emissions facing business environment uncertainty. The study first defined a general closed-loop supply chain scheme, then extended to models that could estimate the logistics expenses with insourcing and outsourcing and their corresponding greenhouse gas emissions. Lastly, the study developed a stochastic model capturing demand uncertainty, facility capacities, used products returning amount and quality, warehousing, transportation, and reprocessing expenses. The researchers applied the optimal non-dominant green supply-chain configurations. They used the Epsilon-constraint methodology to resolve their proposed stochastic programming model to provide decision-makers with crucial information on low-carbon investment. Allaham and Dalalah (2022) optimized the utility maintenance jobs with non- preemptive overtime in commercial building environments where the repairman travels and performs maintenance tasks in various maintenance sites. The researchers aimed to balance the needed technicians, the number of required jobs, and appropriate traveling routes with overtime options. The study built a mixed-integer linear programming model with real industry cases to decide the joint decisions regarding time schedules and routing that minimizes overall overtime, traveling, and labor costs. Their resulting optimal decisions helped reduce workforce and overtime usage. Fadile et al. (2022) studied logistics outsourcing in a developing country's firms, specifically the Moroccan manufacturers using a qualitative approach. The researchers investigated the logistics activities, selectable providers' types, factors, and risks influencing the logistics outsourcing decision, benefits, and consequent performances. As a result, their revealed information to facilitate practitioners in Morocco on logistics outsourcing decision-making and pointed out research limitations and future study directions. Other works (Ciliberto & Panzar, 2011; Farliantog, 2016; Chiu et al., 2020; Gaur et al., 2020; Chiu et al., 2021; Babenko et al., 2022; Chiu et al., 2022; de Carvalho et al., 2022; Porto et al., 2022; Chiu et al., 2023; Farghadani-Chaharsooghi & Karimi, 2023; Chiu et al., 2024; Kazancoglu et al., 2024) investigated the effect of various overtime and subcontracting strategies on the fabricating uptime reduction and response time shortening of manufacturing firms and supply chains.

Furthermore, the production management often looks into product inspection with reworking and multi-shipment policies to meet customers' expectations of product quality and low stock holding level. Biswas and Sarker (2008) derived the optimal batch size for a lean fabrication system involving rework and scrap in a production cycle. The study examined three different detective scrap scenarios, including detective scraps before, during, and after rework. According to the shop floor observations and the scrap/rework scenarios, the researchers developed various single-stage inventory models to study their total cost functions to determine the optimal operating policy of lot size and economical storage of inventories. Giri et al. (2016) examined a stochastic demand, random yield supply chain involving a raw material supplier, a producer, and a retailer. The researchers studied the centralized model as the benchmark case, solved the decentralized model, and obtained the Nash equilibrium solutions. The study used numerical examples to demonstrate their proposed models, obtained solutions, and showed that a composite contract, including buyback, sales rebate, and penalty contracts, can coordinate the supply chain. Banerjee et al. (2022) considered the design of fleet size and vehicle routing zones' partition for the same-day delivery systems aiming to improve system efficiency. The researchers applied continuous approximations to maximize the single-vehicle delivery zone's area by capturing average-case operational performance. They first examined dispatch policies considering depot distance optimization to maximize the daily dispatch number per vehicle. Then, the study demonstrated deriving the optimal fleet sizes from area functions and proposing an efficient service region partition approach. Their research scheme was tested through computational studies and operational simulation. Other works (Persson & Göthe-Lundgren, 2005; Giri & Maiti, 2012; Taleizadeh et al., 2017; Chiu et al., 2020; Yassine, 2020; Larbi Rebaiaia & Ait-kadi, 2021; Martins et al., 2021; Carvalho et al., 2022; Gautam et al., 2022; Taheri & Mirzazadeh, 2022; Chiu et al., 2023; Hossain et al., 2023; Zambrano-Rey et al., 2024) explored the impact of product quality issues, their corresponding quality improvement actions, and various finished goods shipping options on meeting customers' expectations and producers' and supply chains' performance. Little past research has yet to develop a decision model to explicitly determine a manufacturing-delivery policy for a multi-item EPQ system with multi-shipment, quality assurance, overtime, postponement, and external source. This work aims to bridge this research gap.

2. The studied multi-item postponement problem

The studied multi-item postponement problem considered the following: a two-stage delay differentiation manufacturing, subcontracting the stage one's common components, overtime fabricating of stage two's finished goods, quality assurance actions in both phases, and multiple deliveries of finished goods. To illustrate our approach, we first offer Nomenclature (as exhibited in Appendix A) to define all relating symbols and the following problem statement before building our analytical model. A two-stage batch multi-item postponement model with the rotation cycle time discipline is built to meet the L distinct products' annual demand λ_i (where i = 1, 2, …, L). Coping with the commonality in these L different products, we apply a postponement policy to first fabricate all mutual components and make each finished item in the second phase. We also adopt a constant common part's completing rate γ. For example, when $\gamma = 0.5$ (or the common component is 50% completion), then $P_{1,0}$ and $P_{1,i}$ are both twice as much as their standard fabricating rates in a multi-item production model without postponement (i.e., a single-stage multi-item fabricating system). Fig. 1 depicts our model's inventory status, and we can observe the cycle time as follows:

$$T_Z = t_{1,i} + t_{2,i} + t_{3,i} \quad where\ i = 0,\ 1,\ 2,\ \dots,\ L \tag{1}$$

A partial subcontracting policy is implemented in stage one to shorten the uptime for making standard components. An overtime policy is employed in stage two to cut short the needed time for producing the finished products. Consider π_0 as the subcontracting percentage of the common component's batch in phase one and $\alpha_{1,i}$ is the added output rate when applying the overtime policy for producing L different finished products in phase two (where i = 1, 2, …, L). Then, the following formulas exhibit the consequent conversion of cost parameters and fabricating/reworking rates due to subcontracting and overtime strategies (see the nomenclature for the detailed notation definition):

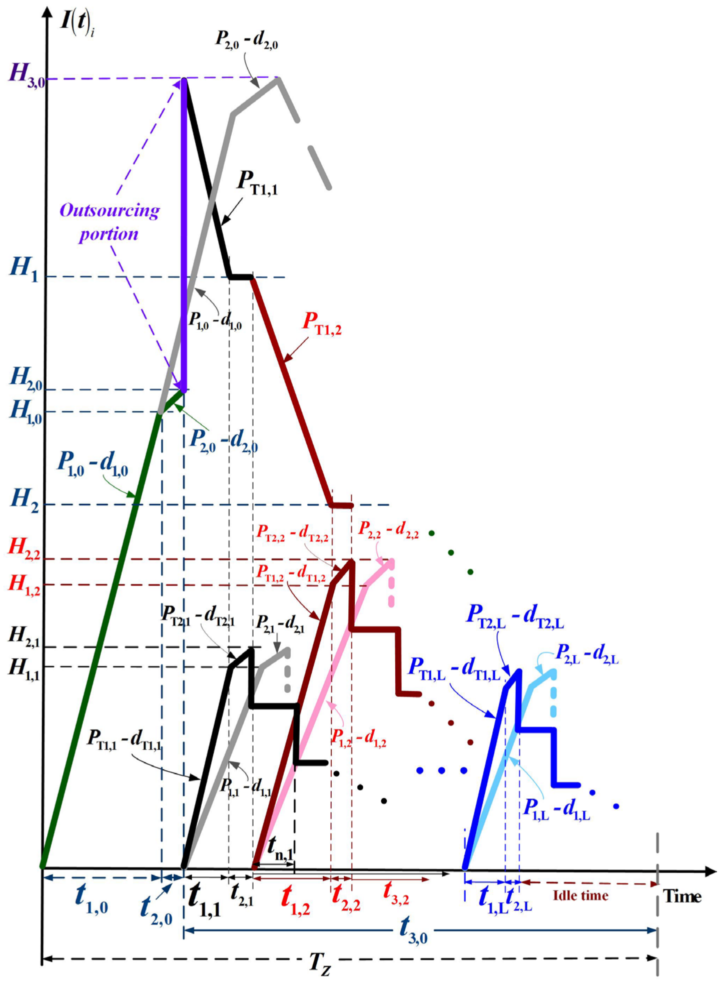

Fig. 1. The studied multi-item postponement problem's stock level in a replenishing cycle compared to the same system without subcontracting and overtime

$$C_{\pi 0} = \left(1+\beta_{2,0}\right)C_0 \tag{2}$$

$$K_{\pi 0} = \left(1+\beta_{1,0}\right)K_0 \tag{3}$$

$$P_{\mathrm{T}1,i} = \left(1+\alpha_{1,i}\right)P_{1,i} \tag{4}$$

$$P_{\mathrm{T}2,i} = \left(1+\alpha_{1,i}\right)P_{2,i} \tag{5}$$

$$K_{\mathrm{T},i} = \left(1+\alpha_{2,i}\right)K_i \tag{6}$$

$$C_{\mathrm{T},i} = \left(1+\alpha_{3,i}\right)C_i \tag{7}$$

$$C_{\mathrm{T}R,i} = \left(1+\alpha_{3,i}\right)C_{R,i} \tag{8}$$

Imperfect fabrication exists in both stages 1 and 2. Consider x_0 and x_i as the random defective rates in each stage. Among the faulty items, scrap portion $\theta_{1,0}$ and $\theta_{1,i}$ is identified and removed before reworking the rest of the faulty items. Observing stage 1 of Fig. 1, one finds when uptime stops, the inventory accumulates to $H_{1,0}$, and it arrives at $H_{2,0}$ when rework finishes. By the predetermined subcontracting arrival schedule, the stock adds up to $H_{3,0}$. No permitting shortage in our study, that is $(P_{1,0} - d_{1,0}) > 0$ and $(P_{\mathrm{T}1,i} - d_{\mathrm{T}1,i} - \lambda_i) > 0$. Meantime, the common component level declines Q_i amount from $H_{3,0}$ to H_i when each finished product uptime ends (see Fig. 1 and Fig. 2).

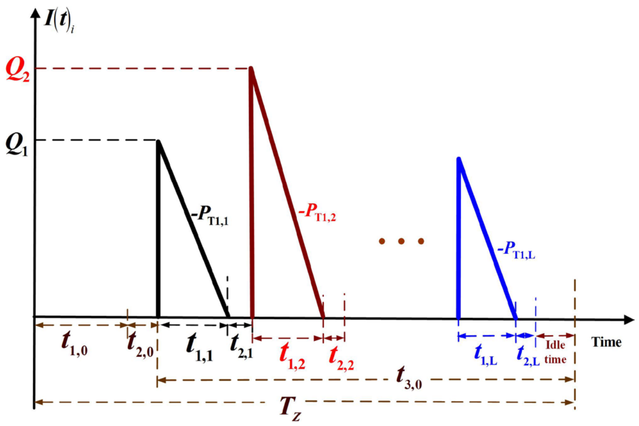

Fig. 2. Common component's level in stage 2 of a cycle

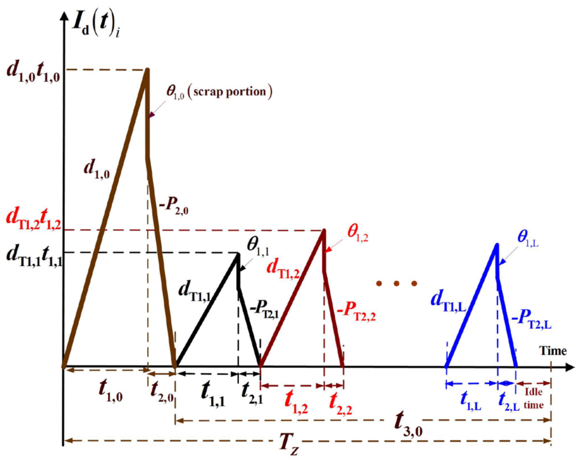

Fig. 3. Random defective items status in our replenishing cycle

Observing stage 2 of Fig. 1, the stock piles up to $H_{1,i}$ when the product i's uptime ends, and it reaches $H_{2,i}$, when its rework ends. Figs 3 and 4 display the random defective and scrap items status in our replenishing cycle.

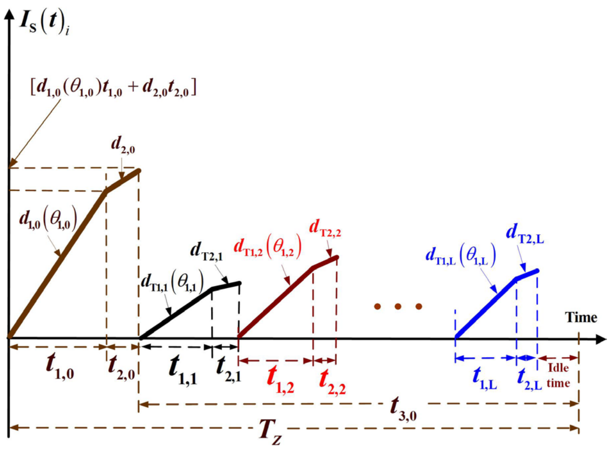

Fig. 4. Random scrap items status in our replenishing cycle

Observing Fig. 3, one finds in the end of phase one's uptime, the random defective stocks reach $(d_{1,0}\, t_{1,0})$. After all random scrap items are removed, the rework process gradually consumes the random faulty items until their level becomes empty. By observing Fig. 4, the highest scrap items arrive at $[d_{1,0}(\theta_{1,0})t_{1,0} + d_{2,0}\, t_{2,0}]$ as phase one's uptime and rework times end. The end product i's scrapped items pile up to a max of $[d_{\mathrm{T}1,i}\,(\theta_{1,i})t_{1,i} + d_{\mathrm{T}2,i}\, t_{2,i}]$ as stage two's uptime and rework times end.

2.1. Formulations of stage 2

By observing Figs. 1 to 4, formulation of stage 2 - making and shipping L different end products is exhibited as follows (where $i = 1, 2, \ldots, L$):

$$t_{1,i} = \frac{Q_i}{P_{T1,i}} \tag{9}$$

$$Q_i = \frac{\lambda_i T_Z}{1-\varphi_i x_i} \tag{10}$$

$$H_{1,i} = \left(P_{T1,i} - d_{T1,i}\right) t_{1,i} \tag{11}$$

$$H_{2,i} = H_{1,i} + \left(P_{T2,i} - d_{T2,i}\right) t_{2,i} \tag{12}$$

$$t_{2,i} = \frac{H_{2,i} - H_{1,i}}{P_{T2,i} - d_{T2,ii}} = \frac{Q_i\left[x_i\left(1-\theta_{1,i}\right)\right]}{P_{T2,i}} \tag{13}$$

$$\varphi_i = \theta_{1,i} + \left(1-\theta_{1,i}\right)\theta_{2,i} \tag{14}$$

$$t_{3,i} = T_Z - \left(t_{1,i} + t_{2,i}\right) \tag{15}$$

Each end item i's stock status during shipping time $t_{3,i}$ is shown in Fig. 5. The total inventories in $t_{3,i}$ are as follows:

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right) H_{2,i}\left(t_{3,i}\right) = \left(\frac{1}{n^2}\right) H_{2,i}\left[\frac{n(n-1)}{2}\right]\left(t_{3,i}\right) = \left(\frac{n-1}{2n}\right) H_{2,i}\left(t_{3,i}\right) \tag{16}$$

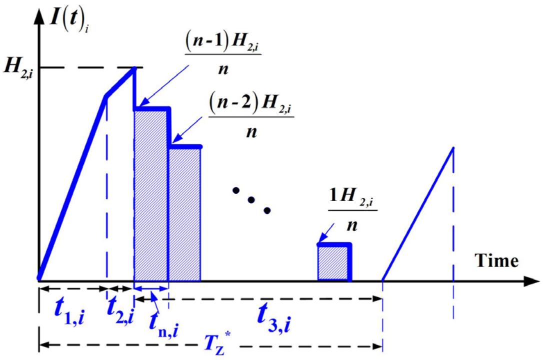

Fig. 5. Each end item i's stock level during shipping time $t_{3,i}$

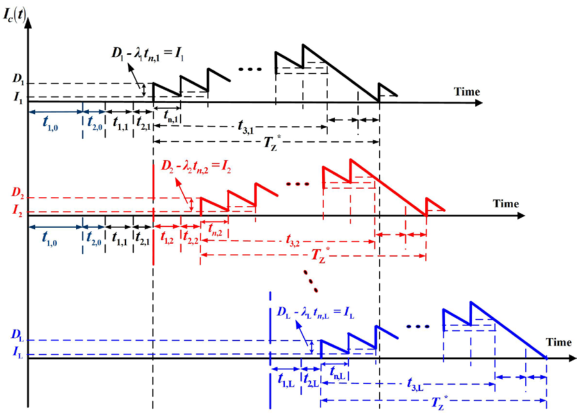

Fig. 6. Stock level of end item i at customer side

Fig. 6 displays the finished products' status on the customer side. Each product i's total inventories are expressed as follows:

$$\left[\frac{n\left(D_i - I_i\right)t_{n,i}}{2} + \frac{n\left(n+1\right)}{2} I_i t_{n,i} + \frac{nI_i\left(t_{1,i}+t_{2,i}\right)}{2}\right] \tag{17}$$

Where

$$D_i = \frac{H_{2,i}}{n} \tag{18}$$

$$I_i = D_i - \lambda_i\left(t_{n,i}\right) \tag{19}$$

$$t_{n,i} = \frac{t_{3,i}}{n} \tag{20}$$

2.2. Formulations of stage 1

Total perfect-quality common components requirements to meet stage two's finished goods needs (Eq. (10)) are as follows:

$$\sum_{i=1}^{L} Q_i = H_{3,0} = \sum_{i=1}^{L} \frac{\lambda_i T_Z}{1-\varphi_i x_i} \tag{21}$$

The common components' annual demand is:

$$\lambda_0 = \frac{\sum_{i=1}^{L} Q_i}{T_Z} \tag{22}$$

With the partial subcontracting strategy, the in-house perfect-quality common components needs are as follows:

$$H_{2,0} = (1-\pi_0) H_{3,0} = (1-\pi_0)\left(\sum_{i=1}^{L} Q_i\right) \tag{23}$$

$$H_{3,0} = H_{2,0} + \pi_0 \left(\sum_{i=1}^{L} Q_i\right) \tag{24}$$

The common components' batch size per cycle Q_0 and related model parameters are exhibited below:

$$Q_0 = \frac{H_{2,0}}{1-\varphi_0 x_0} \tag{25}$$

$$H_{2,0} = H_{1,0} + \left(P_{2,0} - d_{2,0}\right) t_{2,0} \tag{26}$$

$$H_{1,0} = \left(P_{1,0} - d_{1,0}\right) t_{1,0} \tag{27}$$

$$t_{1,0} = \frac{Q_0}{P_{1,0}} \tag{28}$$

$$t_{2,0} = \frac{H_{2,0} - H_{1,0}}{P_{2,0} - d_{2,0}} = \frac{Q_0\left[x_0\left(1-\theta_{1,0}\right)\right]}{P_{2,0}} \tag{29}$$

$$T_Z = t_{1,0} + t_{2,0} + t_{3,0} \tag{30}$$

$$H_1 = H_{3,0} - Q_1 \tag{31}$$

$$H_i = H_{(i-1)} - Q_i \text{ , } for\ i = 2,\ 3,\ \ldots, L \tag{32}$$

$$H_L = H_{(L-1)} - Q_L = 0 \tag{33}$$

$$\varphi_0 = \theta_{1,0} + \left(1-\theta_{1,0}\right)\theta_{2,0} \tag{34}$$

3. The model's cost function & solution procedure

The proposed model's total cost per cycle $TC(T_Z, n)$ comprises (i) stage 1's subcontracting setup and variable expenses, in-house production, disposal, rework, and stock holding expenses; (ii) summation of stage 2's overtime fabricating setup, variable, disposal, rework, stock holding, shipping expenses; and (iii) customer's inventory holding expense. Hence, $TC(T_Z, n)$ becomes as follows:

$$\begin{aligned} TC\left(T_Z,n\right) &= \left[\pi_0\left(\sum_{i=1}^{L} Q_i\right)\right] C_{\pi 0} + K_{\pi 0} + C_0 Q_0 + K_0 + \left[Q_0 x_0\left(1-\theta_{1,0}\right)\right] C_{R,0} + \left(Q_0 x_0 \varphi_0\right) C_{S,0} + h_{4,0}\left(x_0 \varphi_0 Q_0\right) T_Z \\ &+ h_{1,0}\left[\frac{H_{1,0} t_{1,0}}{2} + \frac{d_{1,0} t_{1,0}}{2}\left(t_{1,0}\right) + \frac{H_{2,0}+H_{1,0}}{2}\left(t_{2,0}\right) + \sum_{i=1}^{L}\left[\frac{Q_i}{2}\left(t_{1,i}\right) + H_i\left(t_{1,i}+t_{2,i}\right)\right]\right] + h_{2,0}\left(\frac{d_{1,0} t_{1,0}\left(1-\theta_{1,0}\right)}{2}\right)\left(t_{2,0}\right) \\ &+ \sum_{i=1}^{L} \left\{ \begin{aligned} & Q_i C_{\mathrm{T},i} + K_{\mathrm{T},i} + \left[Q_i x_i\left(1-\theta_{1,i}\right)\right] C_{\mathrm{TR},i} + \left(Q_i x_i \varphi_i\right) C_{S,i} + h_{4,i}\left(x_i \varphi_i Q_i\right) T_Z + h_{2,i}\left(\frac{d_{T1,i} t_{1,i}\left(1-\theta_{1,i}\right)}{2}\right)\left(t_{2,i}\right) \\ & + h_{1,i}\left[\frac{H_{1,i} t_{1,i}}{2} + \frac{d_{T1,i} t_{1,i}}{2}\left(t_{1,i}\right) + \frac{H_{2,i}+H_{1,i}}{2}\left(t_{2,i}\right) + \left(\frac{n-1}{2n}\right) H_{2,i}\left(t_{3,i}\right)\right] + nK_{D,i} + C_{D,i}\left[Q_i\left(1-\varphi_i x_i\right)\right] \\ & + h_{3,i}\left[\frac{n\left(D_i - I_i\right) t_{n,i}}{2} + \frac{n(n+1)}{2} I_i t_{n,i} + \frac{n I_i\left(t_{1,i}+t_{2,i}\right)}{2}\right] \end{aligned} \right\} \end{aligned} \tag{35}$$

Applying $E[x_i]$ (for $i = 0, 1, 2, \dots, L$) to cope with random faulty rates, replace Eqs. (1-34) in $TC(T_Z, n)$, plus additional deriv ation, one derives $E[TCU(T_Z, n)]$ as follows (see Appendix B for details):

$$E\left[TCU\left(T_Z,n\right)\right]=\left\{\begin{array}{l}\frac{\left(1+\beta_{1,0}\right)K_0}{T_Z}+\left(1+\beta_{2,0}\right)C_0\pi_0\lambda_0+C_0\left(1-\pi_0\right)\lambda_0E_{00}+\frac{K_0}{T_Z}+C_{R,0}\left(1-\theta_{1,0}\right)\left(1-\pi_0\right)\lambda_0E_{10}\\ +C_{S,0}\left(1-\pi_0\right)\varphi_0\lambda_0E_{10}+\frac{h_{2,0}\lambda_0^2\left(1-\theta_{1,0}\right)^2\left(1-\pi_0\right)^2}{2P_{2,0}}\left(E_{10}\right)^2T_Z+\frac{h_{1,0}\lambda_0^2T_Z}{2}\left(1-\pi_0\right)^2\left(E_{00}\right)^2E_{0P}\\ +h_{4,0}\left(1-\pi_0\right)\varphi_0\lambda_0E_{10}T_Z+h_{1,0}\sum_{i=1}^{L}\left\{\frac{\lambda_i^2T_Z\left(E_{0i}\right)^2}{2\left[\left(1+\alpha_{1,i}\right)P_{1,i}\right]}+\left(\sum_{i=1}^{L}\left[\lambda_iT_ZE_{0i}\right]-\sum_{j=1}^{i}\left[\lambda_jT_ZE_{0j}\right]\right)\lambda_iE_{0i}E_{2i}\right\}\end{array}\right\}$$

$$+\sum_{i=1}^{L}\left\{\begin{array}{l}\left[\left(1+\alpha_{3,i}\right)C_i\right]\lambda_iE_{0i}+\frac{\left[\left(1+\alpha_{2,i}\right)K_i\right]}{T_Z}+\left[\left(1+\alpha_{3,i}\right)C_{R,i}\right]\left(1-\theta_{1,i}\right)\lambda_iE_{1i}+C_{S,i}\varphi_i\lambda_iE_{1i}+\frac{nK_{D,i}}{T_Z}\\ +C_{D,i}\lambda_i+h_{4,i}\varphi_i\lambda_iE_{1i}T_Z+h_{2,i}\frac{T_Z\left(1-\theta_{1,i}\right)^2}{2\left[\left(1+\alpha_{1,i}\right)P_{2,i}\right]}\left(\lambda_iE_{1i}\right)^2+h_{1,i}\left[\frac{T_Z}{2}\right]\left(\lambda_iE_{0i}\right)^2E_{3i}\\ +h_{1,i}\left[\left(\frac{\lambda_i^2T_Z}{2}\right)E_{3i}\right]+\left(\frac{\lambda_i^2T_Z}{2n}\right)\left(h_{3,i}-h_{1,i}\right)\left[\frac{1}{\lambda_i}-E_{4i}\right]+\frac{h_{3,i}}{2}\left(\lambda_i^2T_Z\right)E_{4i}\end{array}\right\} \tag{36}$$

3.1. The resolution procedure

Hessian Matrix Equations are applied to the $E[TCU(T_Z, n)]$ (Rardin, 1998):

$$\left[T_Z \quad n\right]\cdot\begin{pmatrix}\frac{\partial^2E\left[TCU\left(T_Z,\,n\right)\right]}{\partial T_Z^2} & \frac{\partial^2E\left[TCU\left(T_Z,\,n\right)\right]}{\partial T_Z\partial n}\\ \frac{\partial^2E\left[TCU\left(T_Z,\,n\right)\right]}{\partial T_Z\partial n} & \frac{\partial^2E\left[TCU\left(T_Z,\,n\right)\right]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}T_Z\\ n\end{bmatrix}=\left[\frac{2\left(1+\beta_{1,0}\right)K_0}{T_Z}+\frac{2K_0}{T_Z}+\sum_{i=1}^{L}\left\{\frac{2\left(1+\alpha_{2,i}\right)K_i}{T_Z}\right\}\right]>0 \tag{37}$$

Eq. (37) yields positive, since K_0, $(1 + \beta_{1,0})$, K_i, $(1 + \alpha_{2,i})$, and T_Z are positive. It reconfirms that for all n and T_Z values > 0, $E[TCU(T_Z, n)]$ is strictly convex. Hence, $E[TCU(T_Z, n)]$ has the minimum. Let the first-derivatives of $E[TCU(T_Z, n)]$ concer ning n and T_Z equal to zero, we can simultaneously determine T_Z^* and n^*.

$$\frac{\partial E\left[TCU\left(T_Z,\,n\right)\right]}{\partial n}=\sum_{i=1}^{L}\left\{\frac{K_{D,i}}{T_Z}-\left(\frac{\lambda_i^2T_Z}{2n^2}\right)\left(h_{3,i}-h_{1,i}\right)\left[\frac{1}{\lambda_i}-E_{4i}\right]\right\}=0 \tag{38}$$

$$\frac{\partial E\left[TCU(T_Z,\,n)\right]}{\partial T_Z}=\left\{\begin{array}{l}\frac{-\left(1+\beta_{1,0}\right)K_0}{T_Z^2}-\frac{K_0}{T_Z^2}+\frac{h_{2,0}\lambda_0^2\left(1-\theta_{1,0}\right)^2\left(1-\pi_0\right)^2}{2P_{2,0}}\left(E_{10}\right)^2+\frac{h_{1,0}\lambda_0^2}{2}\left(1-\pi_0\right)^2\left(E_{00}\right)^2E_{0P}\\ +h_{4,0}\left(1-\pi_0\right)\varphi_0\lambda_0E_{10}+h_{1,0}\sum_{i=1}^{L}\left\{\frac{\lambda_i^2\left(E_{0i}\right)^2}{2\left[\left(1+\alpha_{1,i}\right)P_{1,i}\right]}+\left(\sum_{i=1}^{L}\left[\lambda_iE_{0i}\right]-\sum_{j=1}^{i}\left[\lambda_jE_{0j}\right]\right)\lambda_iE_{0i}E_{2i}\right\}\end{array}\right\}$$

$$+\sum_{i=1}^{L}\left\{\begin{array}{l}-\frac{\left[\left(1+\alpha_{2,i}\right)K_i\right]}{T_Z^2}-\frac{nK_{D,i}}{T_Z^2}+h_{4,i}\varphi_i\lambda_iE_{1i}+h_{2,i}\frac{\left(1-\theta_{1,i}\right)^2}{2\left[\left(1+\alpha_{1,i}\right)P_{2,i}\right]}\left(\lambda_iE_{1i}\right)^2\\ +h_{1,i}\left(\frac{\lambda_i^2}{2}\right)E_{3i}\left[1+\left(E_{0i}\right)^2\right]+\left(\frac{\lambda_i^2}{2n}\right)\left(h_{3,i}-h_{1,i}\right)\left[\frac{1}{\lambda_i}-E_{4i}\right]+\frac{h_{3,i}}{2}\left(\lambda_i^2\right)E_{4i}\end{array}\right\}=0 \tag{39}$$

Solving Eqs. (38-39) simultaneously, T_Z^* and n^* are found as follows:

$$T_Z^*=\sqrt{\frac{2\left\{\left(2+\beta_{1,0}\right)K_0+\sum_{i=1}^{L}\left[\left(1+\alpha_{2,i}\right)K_i+nK_{D,i}\right]\right\}}{\begin{array}{l}h_{2,0}\left(E_{10}\right)^2\left(\frac{\left(1-\pi_0\right)^2\lambda_0^2\left(1-\theta_{1,0}\right)^2}{P_{2,0}}\right)+2h_{4,0}\left(1-\pi_0\right)\lambda_0\varphi_0E_{10}\\ +h_{1,0}\left[\left(E_{00}\right)^2\left(1-\pi_0\right)^2\lambda_0^2E_{0P}+\sum_{i=1}^{L}\left[\left(E_{0i}\right)^2\left(\frac{\lambda_i^2}{\left(1+\alpha_{1,i}\right)P_{1,i}}\right)\right]+2\sum_{i=1}^{L}\left(E_{0i}\lambda_i\right)\sum_{i=1}^{L}\lambda_iE_{4i}-2\sum_{i=1}^{L}\left[\left(\sum_{j=1}^{i}\left(E_{0j}\lambda_j\right)\right)\left(\lambda_iE_{4i}\right)\right]\right]\\ +\sum_{i=1}^{L}\left\{h_{2,i}\left(E_{1i}\right)^2\left(\frac{\lambda_i^2\left(1-\theta_{1,i}\right)^2}{\left(1+\alpha_{1,i}\right)P_{2,i}}\right)+h_{1,i}\left(\lambda_i^2\right)E_{3i}+2h_{4,i}\lambda_i\varphi_iE_{1i}+\left(\frac{\lambda_i^2}{n}\right)\left(h_{3,i}-h_{1,i}\right)\left[\frac{1}{\lambda_i}-E_{4i}\right]+h_{3,i}\left(\lambda_i^2\right)E_{4i}\right\}\end{array}}} \tag{40}$$

and

$$n^* = \sqrt{\frac{\left[\left(2+\beta_{1,0}\right)K_0 + \sum_{i=1}^{L}\left(1+\alpha_{2,i}\right)K_i\right] \cdot \sum_{i=1}^{L}\left\{\lambda_i^2\left(h_{3,i}-h_{1,i}\right)\left(\frac{1}{\lambda_i}-E_{4i}\right)\right\}}{\sum_{i=1}^{L}\{2K_{Di}\} \cdot \sum_{i=1}^{L}\left\{\begin{array}{l} h_{2,0}\left(E_{10}\right)^2\left(\frac{\left(1-\pi_0\right)^2\lambda_0^2\left(1-\theta_{1,0}\right)^2}{P_{2,0}}\right) + h_{1,0}\left[\begin{array}{l}\left(E_{00}\right)^2\left(1-\pi_0\right)^2\lambda_0^2E_{0P} + \sum_{i=1}^{L}\left[\left(E_{0i}\right)^2\left(\frac{\lambda_i^2}{\left(1+\alpha_{1,i}\right)P_{1,i}}\right)\right] \\ +2\sum_{i=1}^{L}\left(E_{0i}\lambda_i\right)\sum_{i=1}^{L}\lambda_iE_{4i} - 2\sum_{i=1}^{L}\left[\left(\sum_{j=1}^{i}\left(E_{0j}\lambda_j\right)\right)\left(\lambda_iE_{4i}\right)\right]\end{array}\right] \\ +\sum_{i=1}^{L}\left\{h_{2,i}\left(E_{1i}\right)^2\left(\frac{\lambda_i^2\left(1-\theta_{1,i}\right)^2}{\left(1+\alpha_{1,i}\right)P_{2,i}}\right) + h_{1,i}\left[\lambda_i^2E_{3i}\right] + 2h_{4,i}\lambda_i\varphi_iE_{1i} + h_{3,i}\left(\lambda_i^2\right)E_{4i}\right\} + 2h_{4,0}\left(1-\pi_0\right)\lambda_0\varphi_0E_{10}\end{array}\right\}}} \quad (41)$$

3.2. Sum of setup times discussion

Since a single machine fabricates multiple products, suppose the sum of setup times S_i is larger than the cycle's idle time illustrated in Fig. 1; then, $T_{\min}$ (Nahmias (2009) must be computed. The max of T_Z* (Eq. 40) and $T_{\min}$ should be selected as the cycle length to ensure the machine has sufficient capacity to accommodate the model's required setup and fabricating times.

$$T_{\min} = \frac{\sum_{i=0}^{L}\left(S_i\right)}{1-\left\{\frac{\lambda_0\left(1-\pi_0\right)}{\left[1-\varphi_0E\left[x_0\right]\right]}\left(\frac{1}{P_{1,0}}+\frac{E\left[x_0\right]\left(1-\theta_{1,0}\right)}{P_{2,0}}\right)+\sum_{i=1}^{L}\frac{\lambda_i}{\left[1-\varphi_iE\left[x_i\right]\right]}\left[\frac{1}{P_{T1,i}}+\frac{E\left[x_i\right]\left(1-\theta_{1,i}\right)}{P_{T2,i}}\right]\right\}} \quad (42)$$

3.3. The model's prerequisite condition

Again, as our model uses a single machine to fabricate multiple products, Eq. (43) becomes the prerequisite condition to ensure enough making and reworking capacities in both stages in our model (Nahmias, 2009).

$$\left[\left(t_{1,0}+t_{2,0}\right)+\sum_{i=1}^{L}\left(t_{1,i}+t_{2,i}\right)\right] < T_Z \text{ or } \left[Q_0\left(\frac{1}{P_{1,0}}+\frac{E\left[x_0\right]\left(1-\theta_{1,0}\right)}{P_{2,0}}\right)+\sum_{i=1}^{L}Q_i\left(\frac{1}{P_{T1,i}}+\frac{E\left[x_i\right]\left(1-\theta_{1,i}\right)}{P_{T2,i}}\right)\right] < T_Z \quad (43)$$

or

$$\left\{\left(\frac{\lambda_0\left(1-\pi_0\right)}{\left[1-\varphi_0E\left[x_0\right]\right]}\right)\left(\frac{1}{P_{1,0}}+\frac{E\left[x_0\right]\left(1-\theta_{1,0}\right)}{P_{2,0}}\right)+\sum_{i=1}^{L}\left(\frac{\lambda_i}{\left[1-\varphi_iE\left[x_i\right]\right]}\right)\left[\frac{1}{P_{T1,i}}+\frac{E\left[x_i\right]\left(1-\theta_{1,i}\right)}{P_{T2,i}}\right]\right\} < 1 \quad (44)$$

4. Illustration example

An illustration example shows how our research result derives the optimality of manufacturing- shipment policies T_Z* and n* and investigates crucial characteristics of our studied problem. The assumed parameter values for our multi-item postponement EPQ system with an external source, multi-shipment, quality assurance, and overtime are displayed in Tables 1, 2, and 3. Conversely, Tables C-1 and C-2 in Appendix C exhibit their corresponding parameter values for a single-phase manufacturing scheme.

Table 1

Assumed variable values for phase one of our multi-item postponement EPQ system

φ_0	$C_{S,0}$	$P_{1,0}$	γ	λ_0	δ	x_0	$\beta_{1,0}$	$h_{1,0}$	$\theta_{1,0}$
0.09	$10	120000	0.5	17406	0.5	2.5%	-0.7	$8	0.046
$h_{4,0}$	π_0	$P_{2,0}$	C_0	K_0	$h_{2,0}$	$C_{R,0}$	$\beta_{2,0}$	i_0	$\theta_{2,0}$
$8	0.4	96000	$40	$8500	$8	$25	0.4	0.2	0.046

Table 2

Assumed variable values for phase two of our multi-item postponement EPQ system (1 of 2)

Product i	$\alpha_{3,i}$	$K_{D,i}$	$\alpha_{1,i}$	C_i	$P_{1,i}$	$C_{D,i}$	$\alpha_{2,i}$	λ_i	K_i	i_i	$h_{1,i}$
1	0.25	$1800	0.5	$40	112258	$0.1	0.1	3000	$8500	0.2	$8
2	0.25	$1900	0.5	$50	116066	$0.2	0.1	3200	$9000	0.2	$10
3	0.25	$2000	0.5	$60	120000	$0.3	0.1	3400	$9500	0.2	$12
4	0.25	$2100	0.5	$70	124068	$0.4	0.1	3600	$10000	0.2	$14
5	0.25	$2200	0.5	$80	128276	$0.5	0.1	3800	$10500	0.2	$16

Table 3
Assumed variable values for phase two of our multi-item postponement EPQ system (2 of 2)

Product i	φ_i	$\theta_{2,i}$	$h_{3,i}$	x_i	$C_{S,i}$	$\theta_{1,i}$	$h_{4,i}$	$P_{2,i}$	$C_{R,i}$	$h_{2,i}$
1	0.09	0.046	\$70	2.5%	\$10	0.046	\$8	89806	\$25	\$8
2	0.18	0.094	\$75	7.5%	\$15	0.094	\$10	92852	\$30	\$10
3	0.27	0.146	\$80	12.5%	\$20	0.146	\$12	96000	\$35	\$12
4	0.36	0.200	\$85	17.5%	\$25	0.200	\$14	99254	\$40	\$14
5	0.45	0.258	\$90	22.5%	\$30	0.258	\$16	102621	\$45	\$16

To derive optimality of the manufacturing-shipment solution T_Z* and n*, we first apply our previously obtained formulas (40) and (40) and gain T_Z* = 0.5576 and n* = 4. Then, computing formula (36) with these optimal points of T_Z and n, we find the optimal system expenditure $E[TCU(T_Z*, n*)]$ = \$2,579,176. Fig. 7 and Fig. 8 illustrate $E[TCU(T_Z, n)]$'s behavior vis-à-vis T_Z and n, respectively. These Figures explicitly disclose $E[TCU(T_Z, n)]$ significantly rises as T_Z and n leave their optimal values T_Z* and n*.

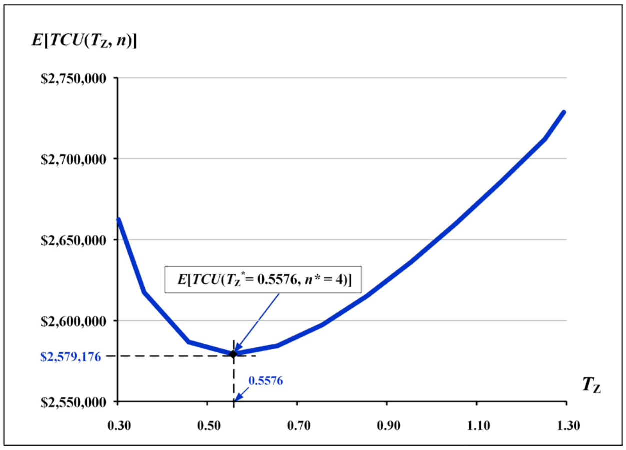

Fig. 7. $E[TCU(T_Z, n)]$'s behavior vis-à-vis T_Z

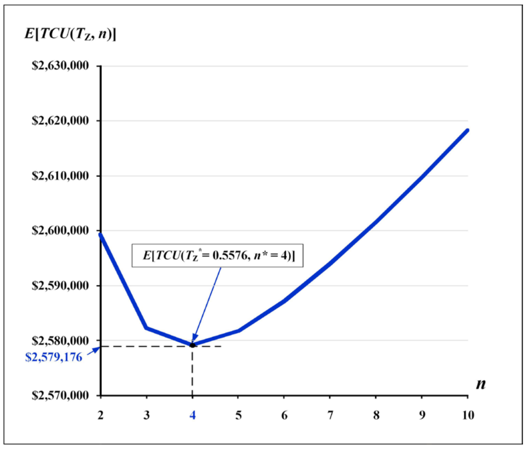

Fig. 8. $E[TCU(T_Z, n)]$'s behavior vis-à-vis n

Our model can investigate the in-depth expenses of the optimal expected system expenditure per unit time $E[TCU(T_Z*, n*)]$. Fig. 9 demonstrates the investigative outcome. It uncovers that the following main expenses add up to 85.81% of $E[TCU(T_Z*, n*)]$:

(a) Finished products' variable cost: 43.05;
(b) Common components' variable expense: 16.43%;
(c) Common components' subcontracting variable cost: 15.29%;
(d) Finished products' overtime expense: 11.04%.

It follows that the supply chain's relevant cost takes up to 6.92%, which includes 3.93% customer holding cost and 2.99% finished products' delivery cost. Then, for both stages, the total setup expense was 3.89%, and the entire quality assurance expense was 3.36%.

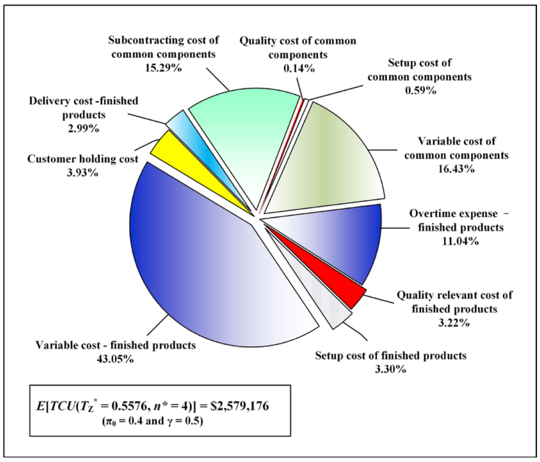

Fig. 9. The investigative outcome of in-depth expenses of $E[TCU(T_Z*, n*)]$

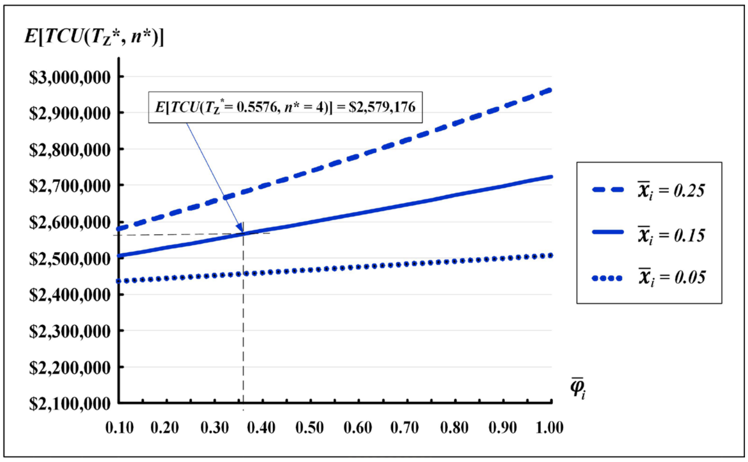

Fig. 10. $E[TCU(T_Z*, n*)]$'s behavior regarding various mean faulty rate and mean scrap rate

The present work is capable of exploring the $E[TCU(T_Z*, n*)]$'s conduct regarding mean scrap rate and various mean faulty rate. As the mean scrap rate rises, more rework task,s and more perfect items need to be made to meet the demand, so $E[TCU(T_Z*, n*)]$ surges knowingly. For the same reason, as the mean faulty fabricating rate increases, $E[TCU(T_Z*, n*)]$ upsurges accordingly.

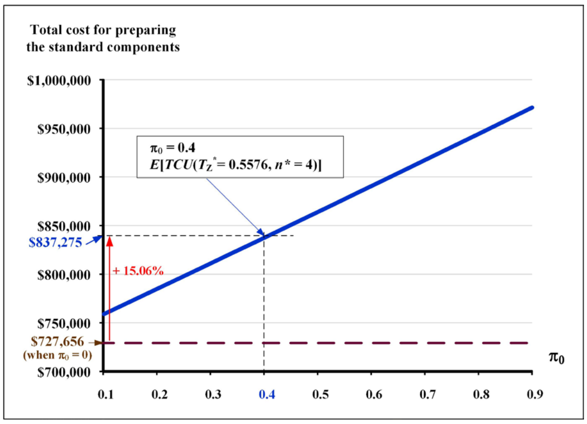

Fig. 11. Total cost for preparing the common components in relation to π_0

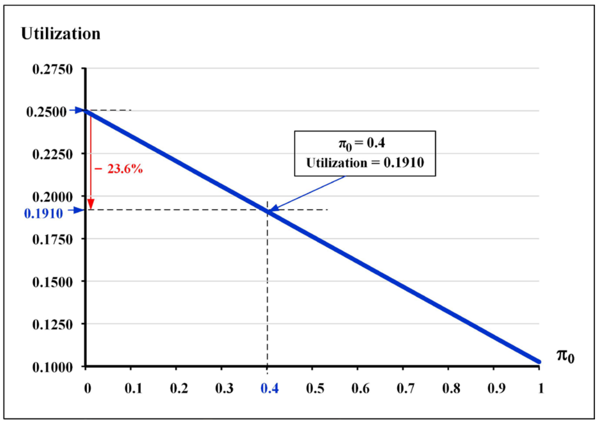

Fig. 12. Utilization's performance in relation to π_0

This work adopts a subcontracting policy to cut stage one's utilization/uptime. The price paid is an extra expenditure since the unit subcontracting cost is higher than the in-house unit cost. Table D-1 (in Appendix D) shows the investigative outcomes of diverse important system parameters impacted by outsourcing portion π_0. Fig. 11 exhibits the explorative outcome of standard components' making expense in relation to subcontracting proportion π_0. As π_0 rises, standard components' expense increases significantly. Additionally, it uncovers that at our assumption $\pi_0 = 0.4$, standard components' expense upsurges a 15.06%, jumping from $727,656 (when $\pi_0 = 0$) to $837,275 (refer to Table D-1). Since the price of extra common components' expense is paid due to subcontracting option for reducing stage one's uptime, one wonders how this influences utilization. Fig. 12 illustrates the investigative result of utilization's performance in relation to subcontracting proportion π_0. As π_0 rises, utilization drops significantly, and it uncovers that at our assumption $\pi_0 = 0.4$, utilization declines a 23.6%, dropping from 0.2500 (when $\pi_0 = 0$) to 0.1910 (see Table D-1). Moreover, one is curious about the increased percentage of $E[TCU(T_Z*, n*)]$ to bring the utilization down by 23.6%, as stated earlier. Fig. 13 exemplifies the investigative outcome of $E[TCU(T_Z*, n*)]$'s performance in relation to subcontracting proportion π_0. As π_0 increases, $E[TCU(T_Z*, n*)]$ surges hugely, and it exposes that at our assumption $\pi_0 = 0.4$, $E[TCU(T_Z*, n*)]$ upsurges a 4.45%, increasing from $2,469,336 (when $\pi_0 = 0$) to $2,579,176 (see Table D-1).

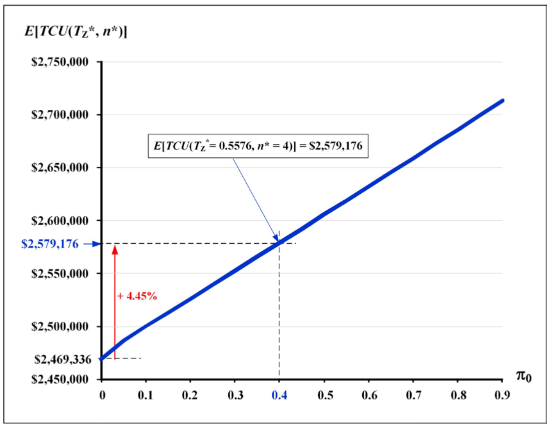

Fig. 13. $E[TCU(T_Z*, n*)]$'s performance in relation to π_0

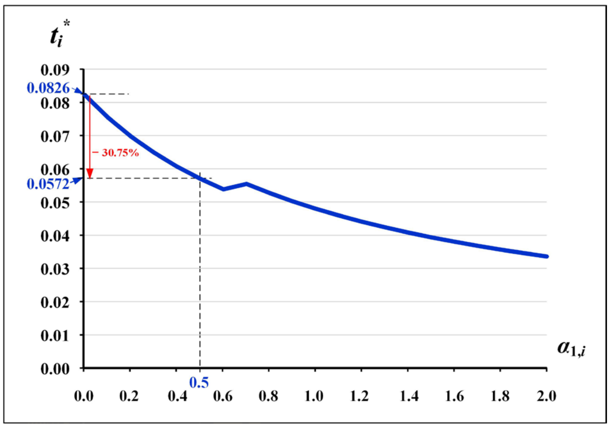

Fig. 14. t_i^*'s performance in relation to $\alpha_{1,i}$

The present work adopts an overtime strategy to make the finished products in stage 2. The overtime option undoubtedly aims to cut phase two's uptime & rework time. One must be curious how the overtime added rate $\alpha_{1,i}$ influences t_i^* (the sum of finished products' optimal uptimes and rework times). Table D-2 (in Appendix D) illustrates the explorative outcomes of diverse important system parameters impacted by overtime ratio $\alpha_{1,0}$. Fig. 14 demonstrates the analytical outcome of t_i^* performance relation to the overtime added rate $\alpha_{1,i}$. As $\alpha_{1,i}$ rises, t_i^* declines brutally. Moreover, it uncovers that at our assumption $\alpha_{1,i} = 0.5$, t_i^* drops a 30.75%, decreasing from 0.0826 (when $\alpha_{1,i} = 0$) to 0.0572. Since the overtime undoubtedly reduces the t_i^*, one wonders how the overtime added rate $\alpha_{1,i}$ influences utilization. Fig. 15 depicts the research outcome of utilization's performance in relation to $\alpha_{1,i}$. As $\alpha_{1,i}$ rises, utilization decreases harshly, and it also discovers that at our assumption $\alpha_{1,i} = 0.5$, utilization drops a 21.2%, declining from 0.2423 (when $\alpha_{1,i} = 0$) to 0.1910 (see Table D-2).

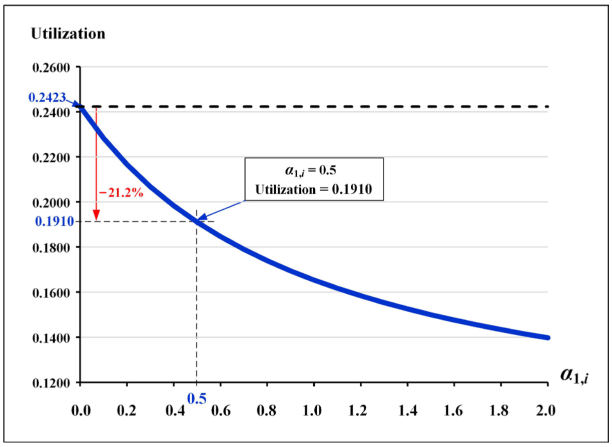

Fig. 15. Utilization's performance in relation to $\alpha_{1,i}$

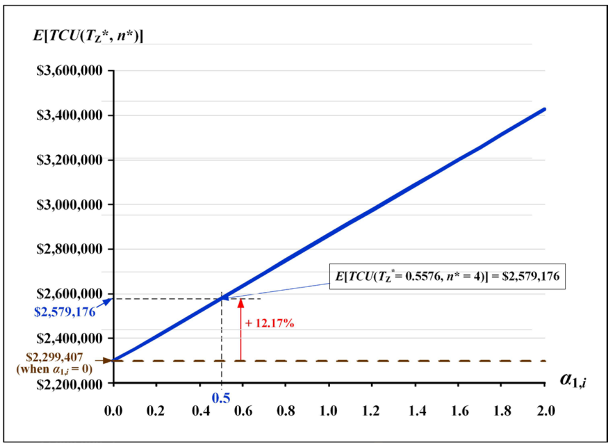

Fig. 16. Performance of $E[TCU(T_Z^*, n^*)]$ relating to $\alpha_{1,i}$

Furthermore, one must be curious about the increased percentage of $E[TCU(T_Z^*, n^*)]$ by the overtime option to bring the utilization down by 21.2%, as stated earlier. Fig. 16 exemplifies the research outcome of $E[TCU(T_Z^*, n^*)]$'s performance in relation to the overtime added output rate $\alpha_{1,i}$. As $\alpha_{1,i}$ rises, $E[TCU(T_Z^*, n^*)]$ surges radically. Besides, it discovers that at our assumption $\alpha_{1,i} = 0.5$, $E[TCU(T_Z^*, n^*)]$ upsurges a 12.17%, rising from \$2,299,407 (when $\alpha_{1,i} = 0$) to \$2,579,176 (refer to Table D-2). This study looks into the incorporated effect of common part's completing rate γ and subcontracting portion π_0 on the optimal expected annual system expense $E[TCU(T_Z^*, n^*)]$. Since subcontracting unit cost is more expensive than in-house unit cost, as π_0 increases, $E[TCU(T_Z^*, n^*)]$ upsurges severely. Especially when γ is higher, there are more loadings on making the common components, and these loads are subcontracted. Fig. 17 exhibits the investigative result of $E[TCU(T_Z^*, n^*)]$'s performance in relation to the combined effect of γ and π_0. As γ increases, $E[TCU(T_Z^*, n^*)]$ marginally decreases when π_0.is low; conversely, when π_0.is high, $E[TCU(T_Z^*, n^*)]$ surges significantly.

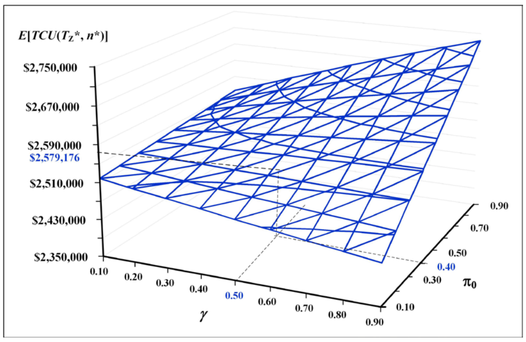

Fig. 17. Performance of $E[TCU(T_Z^*, n^*)]$ relating to the combined impact of γ and π_0

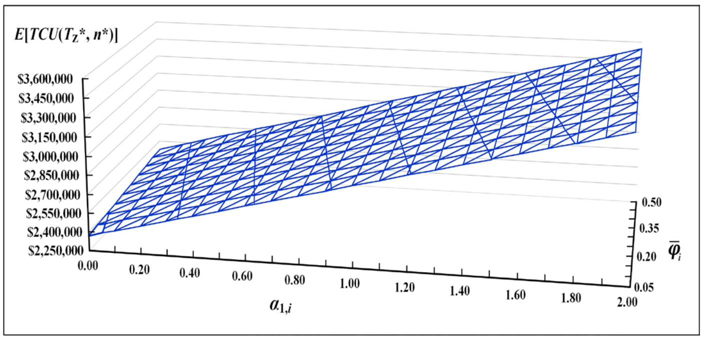

Fig. 18. $E[TCU(T_Z^*, n^*)]$'s conduct regarding the combined impact of $\alpha_{1,i}$ and mean scrap rate

Furthermore, we also study the collective effect of the mean scrap rate and the overtime added output rate $\alpha_{1,i}$. As the mean scrap rate goes up, $E[TCU(T_Z^*, n^*)]$ surges accordingly. Since the overtime unit cost is more expensive than the regular unit cost, as $\alpha_{1,i}$ rises, the optimal expected annual system expense $E[TCU(T_Z^*, n^*)]$ upsurges significantly. Fig. 18 depicts the explorative findings of $E[TCU(T_Z^*, n^*)]$'s performance in relation to the combined impact of $\alpha_{1,i}$ and mean scrap rate. This study adopts subcontracting the common components and overtime fabrication of the end products to shorten utilization; consequently, there are different prices for these strategies, as shown in Figs. (13-16). We further investigate on how to effectively adopt the overtime, subcontracting, or both policies. Fig. 19 demonstrates the findings of $E[TCU(T_Z^*, n^*)]$'s performance concerning these utilization-reduction strategies. The vital message tells decision-makers a cost-effective approach to shorten utilization. That is to subcontract a constant portion $\pi_0 = 0.4$ of the standard components and simultaneously implement overtime with the increasing $\alpha_{1,i}$ (as exhibited in the solid line of Fig. 19). Upon utilization decreases to 0.191 or the overtime rate rises to 0.5 (refer to the intersection of the solid and dashed lines), let $\alpha_{1,i}$ remains unchanged at 0.5 and now start to increase the subcontracting factor π_0. Accordingly, the decision-makers will find the utilization is shortened with a minimal price.

Our illustration example assumes a linear relationship δ between the common component's value and its related completing rate γ, e.g., for $\gamma = 0.5$, the value of the common component is 1/2 of its corresponding end product. Nonetheless, not all merchandise has this linear δ relationship. To further study the impact of this discrepancy on the optimal cycle length of T_Z^*, Fig. 20 demonstrates the further explorative findings of T_Z^*'s performance concerning various relationship δ relating to γ. (e.g., the nonlinear relationship $\delta = \gamma^{1/3}$ and $\delta = \gamma^3$). As γ increases, T_Z^* declines knowingly. It further discovers T_Z^* for the nonlinear relationship $\delta = \gamma^3$ is longer than $\delta = 1$ and $\gamma^{1/3}$.

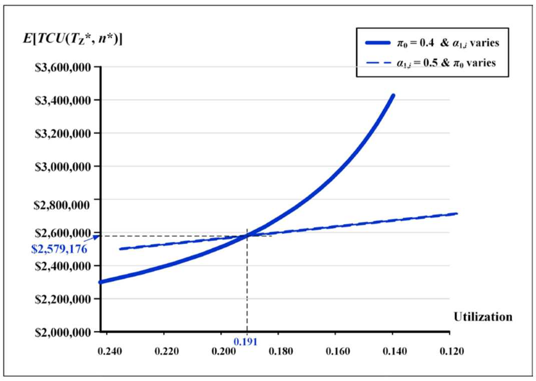

Fig. 19. $E[TCU(T_Z^*, n^*)]$'s performance in relation to the utilization-reduction strategies

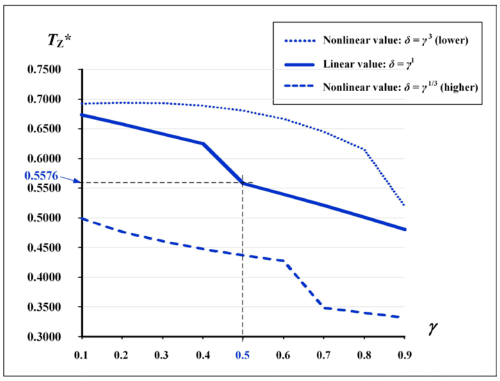

Fig. 20. T_Z^*'s performance concerning various δ relating to γ

5. Conclusions

This work develops a cost-minimization decision model to assist today's producers in meeting present-day client expectations for timely order response, high quality, and multiple shipments of various merchandise. It is a multi-item batch fabrication model and features postponement, dual uptime-reduction strategies, multi-shipment, and quality assurance (see Section 2). The model's overall operating expense consists of in-house setup, quality-relevant (scrapped & reworking), inventory-holding, transportation, contracting-out, and overtime. It is minimized through an optimization methodology to derive the best manufacturing-delivery operating policy (Subsection 3.1). Discussion of the prerequisite requirements for a two-phase multi-item fabrication setup times and capacity are presented (see Subsections 3.2 & 3.2). The capability and applicability of the proposed decision model are illustrated in Section 4 numerically. It shows our decision model not only helps decide the cost-minimization manufacturing-delivery operating policy (Fig. 7 and Fig. 8) but also exposes numerous crucial problem-related insights to facilitate managerial controlling and management in satisfying clients' multi-criteria expectations externally and internal cost-savings (Figs. 9-19). Incorporating the multi-item stochastic demands in our decision model and exploring its influence on the problem solution is an interesting future topic.

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Appendix - A

λ_0 = annual common part's demand rate,
Q_0 = common component's lot size,
t_0^* = sum of common part's optimal uptime plus rework time,
λ_i = annual demand rate of end product i (where i = 1, 2, …, L),
Q_i = finished product i's lot size,
$t_{1,i}$ = finished product i's uptime,
$t_{2,i}$ = finished product i's rework time,
$t_{3,i}$ = finished product i's delivery time,
t_i^* = sum of finished products' optimal uptimes and rework times,
n = equal-size delivery frequency per cycle,
T_Z = fabricating cycle time,
$t_{1,0}$ = common component's fabricating uptime,
$H_{1,0}$ = common component's level when fabricating uptime ends,
$t_{2,0}$ = common component's rework time,
$H_{2,0}$ = common component's level in the end of rework,
$t_{3,0}$ = common component's depleting time,
$H_{3,0}$ = common component's level upon receipt of the subcontract components.
$I(t)_i$ = stock status at time t,
π_0 = common part's subcontracting portion of each lot,
$C_{\pi 0}$ = common component's unit subcontracting cost,
C_0 = common component's unit cost (in-house fabrication),
$\beta_{2,0}$ = linking factor between $C_{\pi 0}$ and C_0,
$K_{\pi 0}$ = common component's fixed subcontracting cost,
K_0 = common component's setup cost (in-house fabrication),
$\beta_{1,0}$ = linking factor between $K_{\pi 0}$ and K_0,
$P_{1,0}$ = common component's annual fabricating rate,
x_0 = common component's random defective rate,
$d_{1,0}$ = defective common component's annual fabricating rate in $t_{2,0}$ (i.e., $d_{1,0} = x_0 P_{1,0}$),
$C_{R,0}$ = common component's unit rework cost,
$\theta_{1,0}$ = not repairable portion of defective common components (prior to rework time),
$h_{1,0}$ = common component's unit holding cost,
$P_{2,0}$ = common part's annual rework rate,
$h_{4,0}$ = safety common component's unit holding cost,
$\theta_{2,0}$ = the failed portion of the reworked common components,
$d_{2,0}$ = scrap common component's annual fabricating rate in $t_{2,0}$ (i.e., $d_{2,0} = \theta_{2,0} P_{2,0}$),
φ_0 = common component's total scrap proportion,
$C_{S,0}$ = scrap common component's unit disposal cost,
$h_{2,0}$ = common component's unit holding cost during rework time,
i_0 = unit holding cost's relating ratio (i.e., $h_{1,0} = i_0 C_0$),
γ = common component's completing rate vs. its finished product,
S_0 = common component's setup time,
H_i = common component's inventory level in the end of end product i,
S_i = finished product i's setup time,
$H_{1,i}$ = finished product i's inventory level when its uptime ends,
$P_{1,i}$ = finished product i's standard annual fabricating rate,
$P_{T1,i}$ = finished product i's annual output rate with overtime implemented,
$\alpha_{1,i}$ = linking factor between $P_{1,i}$ and $P_{T1,i}$,
K_i = finished product i's setup cost,
$K_{T,i}$ = finished product i's setup cost when applying overtime plan,
$\alpha_{2,i}$ = linking factor between K_i and $K_{T,i}$,
C_i = finished product i's unit fabricating cost,
$C_{T,i}$ = finished product i's unit fabricating cost with overtime implemented,

$\alpha_{3,i}$ = linking factor between C_i and $C_{T,i}$,
$h_{1,i}$ = finished product i's unit holding cost,
$h_{4,i}$ = safety finished product i's unit holding cost,
x_i = finished product i's random defective rate,
$d_{T1,i}$ = defective finished product i's fabrication rate (i.e., $d_{T1,i} = x_i P_{T1,i}$),
$\theta_{1,i}$ = not repairable portion of defective end item i (prior to rework time),
$H_{2,i}$ = end product i's inventory level when its rework ends,
$P_{2,i}$ = finished product i's standard annual rework rate,
$P_{T2,i}$ = finished product i's annual rework rate when overtime implemented (i.e., $P_{T2,i} = P_{2,i}(1 + \alpha_{1,i})$),
$C_{TR,i}$ = unit rework cost of end item i when applying overtime,
$C_{R,i}$ = finished product i's standard unit rework cost (i.e., $C_{TR,i} = C_{R,i}(1 + \alpha_{3,i})$),
$C_{S,i}$ = scrap finished product i's unit disposal cost,
$\theta_{2,i}$ = the failed portion of the reworked finished item i,
$d_{T2,i}$ = scrap finished product i's annual fabricating rate in $t_{2,i}$ (i.e., $d_{T2,i} = \theta_{2,i} P_{T2,i}$),
$h_{2,i}$ = unit holding cost for the reworked end product i,
φ_i = end item i's total scrap proportion,
$h_{3,i}$ = customer's unit holding cost,
$I_d(t)_i$ = finished product i's defective inventory level at time t,
$I_S(t)_i$ = scrap level of end product i at time t,
$I_c(t)_i$ = customer side's stock level of end product i at time t,
$t_{n,i}$ = finished product i's fixed time-interval of shippings,
D_i = fixed quantity of finished product i per shipment,
I_i = number of end product i left when $t_{n,i}$ ends,
$K_{D,i}$ = finished product i's fixed delivery cost,
$C_{D,i}$ = finished product i's unit delivery cost,
$E[T_Z]$ = the expected cycle length,
$TC(T_Z, n)$ = total cost in a cycle,
$E[TC(T_Z, n)]$ = expected total cost in a cycle,
$E[TCU(T_Z, n)]$ = the expected annual system cost.

Appendix - B

Detailed derivation of Eq. (36) are as follows:

First, using the expected values $E[x_i]$ (where i = 1, 2, …, L) and $E[x_0]$ to cope with random faulty rates. Then, replacing Eqs. (1) to (34) in $TC(T_Z, n)$ (Eq. (35)) and applying $E[TC(T_Z, n)]/E[T_Z]$ and spending extra efforts in derivation, one derives $E[TCU(T_Z, n)]$ below:

$$E\left[TCU\left(T_Z,n\right)\right]=\left\{\begin{array}{l}
\dfrac{\left(1+\beta_{1,0}\right)K_0}{T_Z}+\left(1+\beta_{2,0}\right)C_{\pi 0}\pi_0\lambda_0+C_0\dfrac{\left(1-\pi_0\right)\lambda_0}{1-\varphi_0 E\left[x_0\right]}+\dfrac{K_0}{T_Z}+C_{R,0}\left[E\left[x_0\right]\left(1-\theta_{1,0}\right)\dfrac{\left(1-\pi_0\right)\lambda_0}{1-\varphi_0 E\left[x_0\right]}\right]\\
+C_{S,0}\left(\dfrac{\left(1-\pi_0\right)\lambda_0 E\left[x_0\right]\varphi_0}{1-\varphi_0 E\left[x_0\right]}\right)+h_{2,0}\left(\dfrac{\left(1-\pi_0\right)^2\lambda_0^2 T_Z E\left[x_0\right]^2\left(1-\theta_{1,0}\right)^2}{2P_{2,0}\left(1-\varphi_0 E\left[x_0\right]\right)^2}\right)+h_{4,0}\left(\dfrac{\left(1-\pi_0\right)\lambda_0 E\left[x_0\right]\varphi_0 T_Z}{1-\varphi_0 E\left[x_0\right]}\right)\\
+h_{1,0}\left[\left[\dfrac{\left(1-\pi_0\right)^2\lambda_0{}^2 T_Z}{2\left(1-\varphi_0 E\left[x_0\right]\right)^2}\right]\left[\dfrac{1}{P_{1,0}}+\dfrac{E\left[x_0\right]\left(1-\theta_{1,0}\right)\left(2-E\left[x_0\right]-\varphi_0 E\left[x_0\right]\right)}{P_{2,0}}\right]+\sum_{i=1}^{L}\left[\dfrac{\lambda_i^2 T_Z}{2\left(1+\alpha_{1,i}\right)P_{1,i}\left(1-\varphi_i E\left[x_i\right]\right)^2}\right]\right]\\
+h_{1,0}\left[\begin{array}{l}
+\displaystyle\sum_{i=1}^{L}\left(\dfrac{\lambda_i T_Z}{1-\varphi_i E\left[x_i\right]}\right)\cdot\sum_{i=1}^{L}\left[\left(\dfrac{\lambda_i}{\left(1+\alpha_{1,i}\right)P_{1,i}\left(1-\varphi_i E\left[x_i\right]\right)}+\dfrac{\lambda_i\left[E\left[x_i\right]\left(1-\theta_{1,i}\right)\right]}{\left(1+\alpha_{1,i}\right)P_{2,i}\left(1-\varphi_i E\left[x_i\right]\right)}\right)\right]\\
+\displaystyle\sum_{i=1}^{L}\left[\left(\sum_{j=1}^{i}\dfrac{\lambda_j T_Z}{1-\varphi_j E\left[x_j\right]}\right)\cdot\left(\dfrac{\lambda_i}{\left(1+\alpha_{1,i}\right)P_{1,i}\left(1-\varphi_i E\left[x_i\right]\right)}+\dfrac{\lambda_i\left[E\left[x_i\right]\left(1-\theta_{1,i}\right)\right]}{\left(1+\alpha_{1,i}\right)P_{2,i}\left(1-\varphi_i E\left[x_i\right]\right)}\right)\right]
\end{array}\right]
\end{array}\right\}$$

$$+\sum_{i=1}^{L}\left\{\begin{array}{l}
\left(1+\alpha_{3,i}\right)C_i\left(\dfrac{\lambda_i}{1-\varphi_i E\left[x_i\right]}\right)+\dfrac{\left(1+\alpha_{2,i}\right)K_i}{T_Z}+C_{TR,i}\left[E\left[x_i\right]\left(1-\theta_{1,i}\right)\left(\dfrac{\lambda_i}{1-\varphi_i E\left[x_i\right]}\right)\right]+C_{S,i}\left(\dfrac{E\left[x_i\right]\varphi_i\lambda_i}{1-\varphi_i E\left[x_i\right]}\right)\\
+h_{2,i}\left(\dfrac{\lambda_i^2 T_Z E\left[x_i\right]^2\left(1-\theta_{1,i}\right)^2}{2\left(1+\alpha_{1,i}\right)P_{2,i}\left(1-\varphi_i E\left[x_i\right]\right)^2}\right)+h_{4,i}\left(\dfrac{E\left[x_i\right]\lambda_i\varphi_i}{1-\varphi_i E\left[x_i\right]}\right)T_Z+\dfrac{nK_{D,i}}{T_Z}+C_{D,i}\lambda_i\\
+h_{1,i}\left[\left(\dfrac{\lambda_i^2 T_Z}{2}\right)\left(\dfrac{1}{\lambda_i}+\dfrac{E\left[x_i\right]\varphi_i}{\left(1+\alpha_{1,i}\right)P_{1,i}\left(1-\varphi_i E\left[x_i\right]\right)^2}+\dfrac{E\left[x_i\right]\left(1-\theta_{1,i}\right)\left(1-E\left[x_i\right]\right)}{\left(1+\alpha_{1,i}\right)P_{2,i}\left(1-\varphi_i E\left[x_i\right]\right)^2}\right)\right]\\
+\left(\dfrac{\lambda_i{}^2 T_Z}{2n}\right)\left(h_{3,i}-h_{1,i}\right)\left[\dfrac{1}{\lambda_i}-\dfrac{1}{\left(1+\alpha_{1,i}\right)P_{1,i}\left(1-\varphi_i E\left[x_i\right]\right)}-\dfrac{E\left[x_i\right]\left(1-\theta_{1,i}\right)}{\left(1+\alpha_{1,i}\right)P_{2,i}\left(1-\varphi_i E\left[x_i\right]\right)}\right]\\
+\dfrac{h_{3,i}}{2}\left(\lambda_i^2 T_Z\right)\left(\dfrac{1}{\left(1+\alpha_{1,i}\right)P_{1,i}\left(1-\varphi_i E\left[x_i\right]\right)}+\dfrac{E\left[x_i\right]\left(1-\theta_{1,i}\right)}{\left(1+\alpha_{1,i}\right)P_{2,i}\left(1-\varphi_i E\left[x_i\right]\right)}\right)
\end{array}\right\} \tag{B1}$$

Let E_{00}, E_{10}, E_{0j}, E_{0P}, E_{0i}, E_{1i}, E_{2i}, E_{3i}, and E_{4i} denote the following:

$$E_{00} = \frac{1}{\left(1-\varphi_0 E[x_0]\right)};\ E_{10} = \frac{E[x_0]}{\left(1-\varphi_0 E[x_0]\right)};\ E_{0j} = \frac{1}{\left(1-\varphi_j E\left[x_j\right]\right)}\ \ for\ \ j=1,\dots,i \tag{B-2}$$

$$E_{0P} = \left[\frac{1}{P_{1,0}} + \frac{E[x_0]\left(1-\theta_{1,0}\right)\left[2-E[x_0]\left(\varphi_0+1\right)\right]}{P_{2,0}}\right].$$

$$E_{0i} = \frac{1}{\left(1-\varphi_i E[x_i]\right)};\ E_{1i} = \frac{E[x_i]}{\left(1-\varphi_i E[x_i]\right)};\ E_{2i} = \left[\frac{1}{\left[\left(1+\alpha_{1,i}\right)P_{1,i}\right]} + \frac{E[x_i]\left(1-\theta_{1,i}\right)}{\left[\left(1+\alpha_{1,i}\right)P_{2,i}\right]}\right]; \tag{B-3}$$

$$E_{3i} = \left[\frac{1}{\lambda_i} + \frac{\left(E_{0i}\right)\left(E_{1i}\right)\varphi_i}{\left(1+\alpha_{1,i}\right)P_{1,i}} + \frac{\left(E_{0i}\right)\left(E_{1i}\right)\left(1-\theta_{1,i}\right)\left(1-E[x_i]\right)}{\left(1+\alpha_{1,i}\right)P_{2,i}}\right]\ \ for\ \ i=1,\dots,L.$$

$$E_{4i} = \left[\frac{E_{0i}}{\left(1+\alpha_{1,i}\right)P_{1,i}} + \frac{E_{1i}\left(1-\theta_{1,i}\right)}{\left(1+\alpha_{1,i}\right)P_{2,i}}\right]\ \ for\ \ i=1,\dots,L. \tag{B-4}$$

Substitute Eqs. (B-2), (B-3), and (B-4) in Eq. (B-1), $E[TCU(T_Z)]$ becomes as follows:

$$\begin{aligned} E\left[TCU\left(T_Z,n\right)\right] = &\left\{\begin{aligned} &\frac{\left(1+\beta_{1,0}\right)K_0}{T_Z} + \left(1+\beta_{2,0}\right)C_0\pi_0\lambda_0 + C_0\left(1-\pi_0\right)\lambda_0 E_{00} + \frac{K_0}{T_Z} + C_{R,0}\left(1-\theta_{1,0}\right)\left(1-\pi_0\right)\lambda_0 E_{10} \\ &+C_{S,0}\left(1-\pi_0\right)\varphi_0\lambda_0 E_{10} + \frac{h_{2,0}\lambda_0^2\left(1-\theta_{1,0}\right)^2\left(1-\pi_0\right)^2}{2P_{2,0}}\left(E_{10}\right)^2 T_Z + \frac{h_{1,0}\lambda_0^2 T_Z}{2}\left(1-\pi_0\right)^2\left(E_{00}\right)^2 E_{0P} \\ &+h_{4,0}\left(1-\pi_0\right)\varphi_0\lambda_0 E_{10}T_Z + h_{1,0}\sum_{i=1}^{L}\left\{\frac{\lambda_i^2 T_Z\left(E_{0i}\right)^2}{2\left[\left(1+\alpha_{1,i}\right)P_{1,i}\right]} + \left(\sum_{i=1}^{L}\left[\lambda_i T_Z E_{0i}\right] - \sum_{j=1}^{i}\left[\lambda_j T_Z E_{0j}\right]\right)\lambda_i E_{0i}E_{2i}\right\} \end{aligned}\right\} \\ &+\sum_{i=1}^{L}\left\{\begin{aligned} &\left[\left(1+\alpha_{3,i}\right)C_i\right]\lambda_i E_{0i} + \frac{\left[\left(1+\alpha_{2,i}\right)K_i\right]}{T_Z} + \left[\left(1+\alpha_{3,i}\right)C_{R,i}\right]\left(1-\theta_{1,i}\right)\lambda_i E_{1i} + C_{S,i}\varphi_i\lambda_i E_{1i} + \frac{nK_{D,i}}{T_Z} \\ &+C_{D,i}\lambda_i + h_{4,i}\varphi_i\lambda_i E_{1i}T_Z + h_{2,i}\frac{T_Z\left(1-\theta_{1,i}\right)^2}{2\left[\left(1+\alpha_{1,i}\right)P_{2,i}\right]}\left(\lambda_i E_{1i}\right)^2 + h_{1,i}\left[\frac{T_Z}{2}\right]\left(\lambda_i E_{0i}\right)^2 E_{3i} \\ &+h_{1,i}\left[\left(\frac{\lambda_i^2 T_Z}{2}\right)E_{3i}\right] + \left(\frac{\lambda_i^2 T_Z}{2n}\right)\left(h_{3,i}-h_{1,i}\right)\left[\frac{1}{\lambda_i} - E_{4i}\right] + \frac{h_{3,i}}{2}\left(\lambda_i^2 T_Z\right)E_{4i} \end{aligned}\right\} \end{aligned} \tag{36}$$

Appendix - C

Table C-1
Corresponding variable values in a single-phase scheme of the same problem (1/2)

Product i	φ_i	λ_i	$h_{1,i}$	K_i	$C_{D,i}$	$P_{1,i}$	C_i	i_i	$h_{4,i}$
1	0.18	3000	$16	$17000	$0.1	58000	$80	0.2	$16
2	0.27	3200	$18	$17500	$0.2	59000	$90	0.2	$18
3	0.36	3400	$20	$18000	$0.3	60000	$100	0.2	$20
4	0.45	3600	$22	$18500	$0.4	61000	$110	0.2	$22
5	0.54	3800	$24	$19000	$0.5	62000	$120	0.2	$24

Table C-2
Corresponding variable values in a single-phase scheme of the same problem (2/2)

Product i	$\theta_{1,i}$	$K_{D,i}$	$C_{R,i}$	$h_{3,i}$	x_i	$C_{S,i}$	$P_{2,i}$	$h_{2,i}$	$\theta_{2,i}$
1	0.094	$1800	$50	$70	5%	$20	46400	$16	0.094
2	0.146	$1900	$55	$75	10%	$25	47200	$18	0.146
3	0.200	$2000	$60	$80	15%	$30	48000	$20	0.200
4	0.258	$2100	$65	$85	20%	$35	48800	$22	0.258
5	0.322	$2200	$70	$90	25%	$40	49600	$24	0.322

Appendix – D

Table D-1
Diverse important system parameters impacted by outsourcing portion π_0

π_0	Utilization (A)	(A) decline %	t_0*	t_0* drop%-	E[$TCU(T_Z*,n*)$] (B)	(B)% surge	Extra expense due to outsourcing	Quality cost in stage 1	Total cost for preparing common components	T_Z*	n*
0.00	25.00%	-	0.0814	-	$2,469,336	-	$0	$6,176	$727,656	0.5452	4
0.05	24.26%	-2.95%	0.0774	-4.86%	$2,487,034	0.72%	$53,349	$5,867	$745,237	0.5529	4
0.10	23.52%	-5.90%	0.0734	-9.73%	$2,500,112	1.25%	$102,079	$5,558	$758,299	0.5537	4
0.15	22.79%	-8.84%	0.0695	-14.63%	$2,513,219	1.78%	$150,809	$5,249	$771,390	0.5545	4
0.20	22.05%	-11.79%	0.0655	-19.55%	$2,526,353	2.31%	$199,540	$4,941	$784,509	0.5552	4
0.25	21.31%	-14.74%	0.0614	-24.49%	$2,539,516	2.84%	$248,272	$4,632	$797,657	0.5559	4
0.30	20.57%	-17.69%	0.0574	-29.44%	$2,552,708	3.38%	$297,003	$4,323	$810,834	0.5565	4
0.35	19.84%	-20.64%	0.0534	-34.41%	$2,565,928	3.91%	$345,735	$4,014	$824,040	0.5571	4
0.40	**19.10%**	**-23.58%**	**0.0493**	**-39.39%**	**$2,579,176**	**4.45%**	**$394,467**	**$3,705**	**$837,275**	**0.5576**	**4**
0.45	18.36%	-26.53%	0.0452	-44.39%	$2,592,454	4.99%	$443,200	$3,396	$850,540	0.5581	4
0.50	17.63%	-29.48%	0.0412	-49.41%	$2,605,760	5.52%	$491,933	$3,088	$863,834	0.5586	4
0.55	16.89%	-32.43%	0.0371	-54.43%	$2,619,095	6.06%	$540,666	$2,779	$877,158	0.5590	4
0.60	16.15%	-35.38%	0.0330	-59.47%	$2,632,459	6.61%	$589,400	$2,470	$890,512	0.5594	4
0.65	15.42%	-38.32%	0.0289	-64.51%	$2,645,852	7.15%	$638,134	$2,161	$903,896	0.5598	4
0.70	14.68%	-41.27%	0.0248	-69.57%	$2,659,274	7.69%	$686,868	$1,853	$917,310	0.5601	4
0.75	13.94%	-44.22%	0.0206	-74.63%	$2,672,725	8.24%	$735,603	$1,544	$930,754	0.5603	4
0.80	13.21%	-47.17%	0.0165	-79.69%	$2,686,205	8.78%	$784,338	$1,235	$944,229	0.5605	4
0.85	12.47%	-50.12%	0.0124	-84.77%	$2,699,714	9.33%	$833,074	$926	$957,734	0.5607	4
0.90	11.73%	-53.06%	0.0083	-89.84%	$2,713,253	9.88%	$881,810	$617	$971,269	0.5608	4
0.95	11.00%	-56.01%	0.0041	-94.92%	$2,726,821	10.43%	$930,546	$309	$984,835	0.5609	4
1.00	10.26%	-58.96%	0.0000	-100.0%	$2,724,938	10.35%	$979,482	$0	$983,308	0.5373	4

Table D-2
Diverse important system parameters impacted by overtime ratio $\alpha_{1,0}$

$\alpha_{1,0}$	Utilization (A)	(A) decline %	Sum of $t_{1,i}$* (B)	(B) drop%-	E[$TCU(T_Z*,n*)$] (C)	(C)% surge	Extra expense due to Overtime	Finished products shipping expense	Quality cost in stage 2	T_Z*	n*
0.0	24.23%	-	0.0826	-	$2,299,407	-	$0	$180,362	$76,085	0.5370	4
0.1	22.83%	-5.77%	0.0758	-8.30%	$2,355,028	2.42%	$55,932	$179,803	$77,949	0.5417	4
0.2	21.67%	-10.58%	0.0700	-15.27%	$2,410,858	4.85%	$111,836	$179,359	$79,818	0.5460	4
0.3	20.68%	-14.66%	0.0651	-21.21%	$2,466,848	7.28%	$167,715	$179,003	$81,690	0.5501	4
0.4	19.83%	-18.14%	0.0609	-26.32%	$2,522,963	9.72%	$223,572	$178,717	$83,566	0.5539	4
0.5	**19.10%**	**-21.17%**	**0.0572**	**-30.75%**	**$2,579,176**	**12.17%**	**$279,408**	**$178,488**	**$85,448**	**0.5576**	**4**
0.6	18.46%	-23.81%	0.0540	-34.69%	$2,635,469	14.62%	$335,225	$178,306	$87,327	0.5612	4
0.7	17.89%	-26.15%	0.0556	-32.66%	$2,693,960	17.16%	$390,063	$177,041	$89,222	0.6148	5
0.8	17.39%	-28.22%	0.0529	-36.04%	$2,750,170	19.60%	$445,716	$176,750	$91,104	0.6183	5
0.9	16.94%	-30.08%	0.0503	-39.07%	$2,806,430	22.05%	$501,355	$176,495	$92,986	0.6217	5
1.0	16.54%	-31.75%	0.0481	-41.81%	$2,862,732	24.50%	$556,980	$176,271	$94,869	0.6250	5
1.1	16.17%	-33.26%	0.0460	-44.29%	$2,919,071	26.95%	$612,592	$176,074	$96,754	0.6283	5
1.2	15.84%	-34.64%	0.0442	-46.55%	$2,975,439	29.40%	$668,191	$175,901	$98,639	0.6314	5
1.3	15.53%	-35.89%	0.0425	-48.63%	$3,031,833	31.85%	$723,778	$175,748	$100,524	0.6345	5
1.4	15.25%	-37.04%	0.0409	-50.53%	$3,088,248	34.31%	$779,353	$175,614	$102,411	0.6376	5
1.5	15.00%	-38.10%	0.0394	-52.29%	$3,144,682	36.76%	$834,916	$175,497	$104,298	0.6406	5
1.6	14.76%	-39.08%	0.0381	-53.91%	$3,201,132	39.22%	$890,468	$175,394	$106,185	0.6436	5
1.7	14.54%	-39.98%	0.0368	-55.41%	$3,257,595	41.67%	$946,009	$175,305	$108,072	0.6465	5
1.8	14.34%	-40.82%	0.0357	-56.82%	$3,314,070	44.13%	$1,001,539	$175,228	$109,960	0.6494	5
1.9	14.15%	-41.61%	0.0346	-58.12%	$3,370,555	46.58%	$1,057,059	$175,162	$111,849	0.6522	5
2.0	13.97%	-42.34%	0.0336	-59.34%	$3,427,048	49.04%	$1,112,569	$175,106	$113,737	0.6550	5