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Coordination and optimization decision of assembly building supply chain under supply disruption risk

Zheng Liu[a], Na Huang[a*], Qingshan Qian[a], Yuanjun Zhao[b], Tianchen Yang[a] and Chunjia Han[c]

[a]*School of Management, Shanghai University of Engineering Science, Shanghai 201620, China*
[b]*School of Accounting, Nanjing Audit University, Nanjing 211815, China*
[c]*Department of Management, Birkbeck, University of London, London, WC1E 7HX, United Kingdom*

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A B S T R A C T

Assembly buildings, in the context of the low-carbon transformation of the construction industry, achieve superior outcomes in terms of carbon emission reduction, enhancement of building uniformity, and optimization of resource utilization as compared to traditional buildings. However, the supply chain for assembly building is marked by a significant susceptibility to risk and a need for timely fulfillment of requirements. This paper examines the risk of disruption and capacity limitations in the assembly building supply chain resulting from supply disruptions. It establishes a three-tier supply chain for assembly buildings, including primary component suppliers, backup suppliers, assembly manufacturers, and retailers. The study compares the optimal decision-making and coordination strategies of the supply chain members under centralized, decentralized, and joint agreements. The supply chain dual-source procurement decision coordination model is constructed by incorporating capacity constraints and analyzing the effects of supply disruption probability, repurchase coefficient, revenue sharing coefficient, cost, and other parameters on the expected profits of the supply chain members using arithmetic simulation. Research has indicated that when the likelihood of a disturbance occurring rises, the anticipated financial gain for the main provider decreases, while the predicted financial gain for the secondary supplier increases. The implementation of a collaborative agreement between the assembly maker and the parts backup provider would result in much greater anticipated profits compared to the decentralized decision-making approach. The impact of the revenue sharing coefficient on the predicted earnings of retailers and assembly manufacturers is more significant compared to the repurchase coefficient. The selection bias between NA and NB techniques under capacity constraints mostly arises from the assertiveness of the wholesale asking prices of inexpensive component suppliers, leading assembly manufacturers to increasingly prefer the NA option. This paper's research successfully achieves the contractual coordination of the assembly building supply chain, enhances the resilience of the assembly building supply chain, and promotes the long-term sustainable development of the assembly building supply chain.

1. Introduction

Supply disruption risk typically refers to rare events that have a significant impact and can disrupt the smooth functioning of the supply chain. These events are unpredictable, challenging to identify and assess, and can result in negative consequences, both in the short and long term (Ekanayake et al., 2022). An effective supply chain system necessitates the capacity to endure the possibility of interruption and to recuperate from interruptions with minimal expenses (Massari & Giannoccaro, 2021). Supply chain stability is crucial in the current international supply chain landscape (Shebeshe & Sharma,2024; Sudan et al., 2023). However, supply chain disruptions may have serious impacts on the assembly supply chain, such as negative consequences such as decreased profits (Carvalho et al., 2021), increased costs (Meyer et al., 2023), and impaired productivity

* Corresponding author
E-mail m330122501@sues.edu.cn (N. Huang)
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(Nakano & Lau, 2020) due to supply chain disruptions. Minimizing the likelihood of supply chain interruptions and enhancing the ability of supply chains to recover quickly are crucial for the long-term and robust growth of supply chains. When studying ways to improve supply chain resilience, researchers have found that single or multiple sourcing strategies (Namdar et al., 2018), cooperative strategies (Shekarian & Mellat, 2021), multidimensional flexibility strategies (Piprani et al., 2022), and standby sourcing and backordering strategies (Chen et al., 2024), and optimized supply chain network design (Rezaei & Liu, 2024) have shown better results in enhancing supply chain resilience. By using a diversified sourcing strategy, firms can mitigate the risk associated with depending solely on one supplier and enhance their capacity to manage supply disruptions. Strengthening cooperative strategies, such as establishing long-term relationships with suppliers and implementing supply chain integration (Basana et al., 2024), can improve the overall elastic coordination and responsiveness of the supply chain. Blockchain technology (Ekinci et al., 2024) is utilized to enhance supply chain transparency and traceability, facilitating the prompt identification and response to supply chain risks. The increased adaptive strategy improves the resilience of the entire supply chain system by enhancing the adaptive capabilities of each component in the supply chain (Aboutorab et al., 2024). Complex network theory can be applied to assess and optimize the topology of the supply chain network. It can also help identify nodes that pose potential risks, enabling the development of more effective risk management strategies (Wang et al., 2023). When studying the factors that cause supply chain disruptions, it is important to consider both the common factors like strikes, natural disasters, or equipment failures (Liu & Ren, 2024), as well as supply risk factors that can disrupt a single supplier, and environmental risks that can disrupt coordination among multiple suppliers, ultimately leading to supply disruptions (Kamalahmadi et al., 2017). Supply chain disruptions are more likely to occur when suppliers have internal complexity and when there is complexity in working with other suppliers (Wissuwa et al., 2022). In green building supply chains, key risk factors include technical expertise, skilled labor, key customers, and corporate culture (Marandi et al., 2023). Supply chain disruption is the result of a variety of factors and the external environment, supply chain disruption risk control needs to be carried out from various aspects.

There is a possibility of supply interruption in the supply chain for assembled buildings. The assembly building supply chain is crucial for promoting the process of decarbonization. However, it is highly vulnerable due to the need for timely and well-coordinated projects. Ensuring the stable operation of the supply chain is essential to guaranteeing that construction projects are completed on time and with high quality. Previous researchers have demonstrated that assembly-transportation orientation, planning-control orientation, and manufacturing orientation are the three pathways that result in the development of high-risk assembly supply chains (Wang et al., 2024). Additional analysis of the pathways responsible for creating high-risk supply chains indicates that the speed at which decision makers recognize and comprehend potential supply chain risks is influenced by their perception of risk (Liu & Liu, 2023). Furthermore, the degree of supply chain progress planning and workflow improvement (Luo et al., 2019), and the degree of internal information sharing (Wang et al., 2020) are closely related to the coordination and effectiveness of the whole supply chain. The competence to foresee the risk of disruption has a direct influence on an enterprise's capacity to prevent and prepare for potential disruptions in its supply chain in the future. Forecasting using data analytics (Brintrup et al., 2020) and other techniques allows companies to plan ahead and reduce potential losses. The disruption response time refers to the duration it takes for a company to restore its regular operations following a supply chain disruption. A prompt and efficient response mechanism can significantly mitigate the impact of a disruption event on the supply chain (Zhang et al., 2023). In the realm of prefabricated structures, these characteristics hold significant importance due to the stringent time constraints of projects. Collectively, they constitute the essential elements for enhancing the resilience of the supply chain and guaranteeing timely project completion. Consequently, any disruptions in the supply chain that cause delays or failures at any stage of the process can result in schedule disruptions throughout the entire project. This includes the procurement and production of materials, transportation, and installation, all of which necessitate precise time management and efficient coordination. To reduce this risk, the assembly construction industry is working to implement more sophisticated technologies and methodologies, such as building information modeling. This aims to enhance supply chain resilience and minimize the likelihood of disruptions by influencing participant and partner factors within the supply chain (Hua et al., 2023). Enhancing the resilience of the assembly building supply chain can be achieved by implementing collaborative multi-source sourcing decisions, as discussed in the study by Gurbuz et al. (2023). This approach helps reduce supply latency, improve component quality and standards, as highlighted by Su et al. (2023). Additionally, incorporating business continuity management practices, as suggested by Guntuka et al. (2024), is another effective strategy.

In summary, the literature extensively examines supply chain resilience, factors that influence supply chain disruption, and the disruption of assembly building supply chains. However, the focus of these studies primarily revolves around the influencing factors in the face of supply disruption risks, adjustments in collaborative decision-making, and the utilization of intelligent technologies. The existing research on optimizing the supply chain for constructed buildings has primarily concentrated on analyzing risk factors, predicting disruptions, and developing management models. However, there is a lack of literature that explores both procurement models and contractual coordination solutions for addressing capacity limits during supply disruptions. This study presents a dual-source procurement decision model that takes into account the coordinated optimization strategy of the assembly building supply chain. It considers centralized, decentralized, and joint contracts in the context of supply disruption scenarios. Using this enhanced model, we examine how capacity limitations affect the optimization strategy. We also explore the influence of various factors, such as the likelihood of supply disruptions, the repurchase coefficient, the revenue sharing coefficient, and costs, on the overall supply chain and the expected profit of each participant. Our goal is to achieve contractual coordination in the assembly construction supply chain, leading to Pareto optimality. Additionally, we aim to gain management insights that can enhance the resilience of the assembly supply chain.

2. Model assumptions and parameter description

The assembly building supply chain follows a basic coordination model that comprises three tiers. These tiers include a primary supplier responsible for providing assembly components, a backup supplier, an assembly maker, and a retailer (as shown in Fig. 1). There are two suppliers who offer products or services of equal quality. The main supplier has the advantage of lower wholesale unit prices due to their large scale, but there is a risk of unpredictable supply disruptions. On the other hand, the backup supplier has a consistent and reliable supply of products, although their wholesale unit price is higher. Typically, assembly manufacturers initially purchase components from their main supplier, such as steel structures, wall panels, and other materials for building assemblies. If the main supplier cannot fulfill the order quantity or there is a risk of supply disruption, the assembly manufacturer will turn to their backup supplier to meet the demand of the downstream market. To enhance the enthusiasm of backup suppliers and minimize losses due to out-of-stock situations, the assembly manufacturer implemented a revenue-sharing agreement with backup suppliers. This agreement, based on dual-source procurement, encourages backup suppliers to maintain sufficient stock. Additionally, the assembly manufacturer negotiated a buyback contract with downstream retailers, specifying a repurchase price. This contract reduces retailers' losses caused by stockpiling in case of supply disruptions. Consequently, retailers' concerns regarding ordering are alleviated, leading to improved overall profitability of the supply chain. Optimize the total profit generated by the supply chain. Capacity constraints are a prevalent issue in supply chain enterprises, resulting from various factors such as intricate production processes, inadequate liquidity reserves, insufficient raw material supply, and bottlenecks in key technologies. Ultimately, these constraints manifest as limited production capacity. The study examines the supplier allocation scheme of an assembly manufacturer in a supply chain with two component suppliers (A, B) under capacity constraints. The investigation focuses on the assembly manufacturer's procurement option of dual-sourcing, considering both the risk of supply disruption and the capacity constraints of the component suppliers. A five-stage dynamic decision-making model is established to evaluate the selection of low-cost suppliers for major purchasing decisions (NB strategy) and high-cost suppliers for significant purchasing decisions (NA strategy). In contrast to the standard paradigm, let's imagine that an assembly manufacturer procures core components of identical quality from two upstream component suppliers (A, B). The assembly manufacturer maintains enduring partnerships with both suppliers and does not have any purchasing biases. However, there is a notable disparity in the production capacity of the two suppliers, with x and φ_B being the respective capacities. In contrast to the base model, where the primary supplier is chosen based solely on lower production cost and wholesale price, the assembly manufacturer, operating under capacity constraint, follows a different approach. It first identifies both the primary and backup suppliers. The specific decision-making process for dual-source procurement consists of six distinct stages, as illustrated in Fig. 2. Table 1 displays the symbols representing parameters relevant to the model.

Table 1
Description of model symbols and definitions

Symbol	Meaning	Symbol	Meaning
q_h	Deliveries from major suppliers to assembly manufacturers	s	Retailer out-of-stock costs
q_i	Deliveries from backup suppliers to assembly manufacturers	v	Residual value of unsold products
w_h	Prices of supplies from major suppliers to assembly manufacturers	γ	Buyback price factor offered to retailers by assembly manufacturers $(0<\gamma<1)$
w_i	Supply price from the backup supplier to the assembly manufacturer	κ	Gainsharing factor
w_e	Wholesale prices from assembly manufacturers to downstream retailers	θ	Probability of possible disruptions at major suppliers $(0<\theta<1)$
c_h	Unit cost of spare parts from major suppliers	$f(x)$	Market demand probability density function
c_i	Backup supplier unit cost of spare parts	$F(x)$	Cumulative distribution function of randomized market demand $F(0)=0, F(+\infty)=1$
p	Retailer's selling price	Q	Retailer orders
r_A	Supply disruption risk for Supplier A under production constraints	r_B	Supply disruption risk for Supplier B under production constraints
h	Supplier reliability factor under capacity constraints	D	Potential market demand
$Q_{\&}$	Number of products placed on the market	c_A	Unit production cost of supplier A as a master supplier
c_B	Unit production cost of supplier A as a master supplier	c	Unit production costs in emergency procurement under capacity constraints

To align with the dual-source procurement operation practice and examine the coordination and optimization strategy of the assembly building supply chain, the following hypotheses are formulated for the fundamental coordination model:

Assumption 1: Members of the assembly building supply chain are perfectly rational and hold risk-neutral attitudes.

Assumption 2: $w_e > w_i > w_h > v$, guaranteed profits for assembly manufacturers, component suppliers.

Assumption 3: $p > w_e$, $\gamma w_e > v$, Ensure the validity of repurchase covenants and retailer profits.

Assumption 4: The assembly manufacturer is dominant, and the parts backup supplier has sufficient capacity to meet customer demand.

The dual-source procurement decision coordination model, which takes into account supplier capacity restrictions, proposes the following assumptions:

Assumption 5: The primary supplier commences regular production upon receiving orders; however, the supplier is susceptible to supply disruptions as a result of internal and external random factors. These disruptions are not complete, but the supplier is still partially capable of producing. The fallback supplier is not at risk of supply disruption due to the fact that it is not currently scheduled for regular production and has ample time for equipment maintenance and overhaul, as well as more stable personnel. Considering that the main supplier of parts may be either Supplier A or Supplier B, it is assumed that the probability of their supply disruption risk is r_A and r_B, respectively, and that r_A and r_B, follow a uniform distribution on [0,h] ($0 \le h \le 1$). h is the reliability coefficient, the lower the probability of disruption risk, the more reliable and stable the main supplier of components.

Assumption 6: The assembly manufacturer faces a market inverse demand function of $p = D - Q_{\&}$, where p is the selling price of the product, D is the market potential demand (customer saturation), and $Q_{\&}$ is the number of products placed in the market.

Assumption 7: The unit cost of production of parts suppliers A and B as primary suppliers in regular purchasing is c_A, c_B, while the unit cost of production of either supplier A or B as backup suppliers in emergency purchasing is c, and $c > c_A > c_B$. It is guaranteed that the production cost of component supplier A is higher than that of B, i.e., A is a high-cost component supplier and B is a low-cost component supplier, and it is assumed that the back-up emergency production cost is always higher than the regular production cost to reflect the emergency procurement.

Assumption 8: $\forall i \in \{A, B\}$,with $\varphi_i + c < D < 4\varphi_i + c$. If there is $D \ge 4\varphi_i + c$, it indicates that there is insufficient capacity, and more suppliers need to be deployed. If there is $D < \varphi_i + c$, it indicates that there is excess capacity, and a single supplier can be deployed to satisfy the demand of the small-scale market.

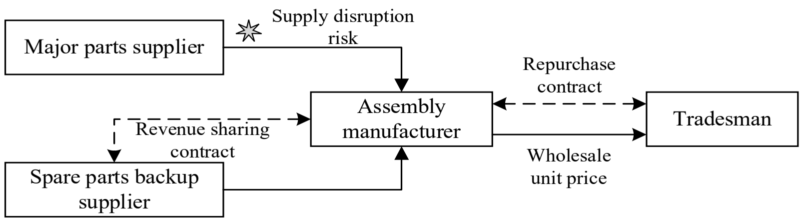

Fig. 1. Assembly supply chain with joint contracts under supply disruption

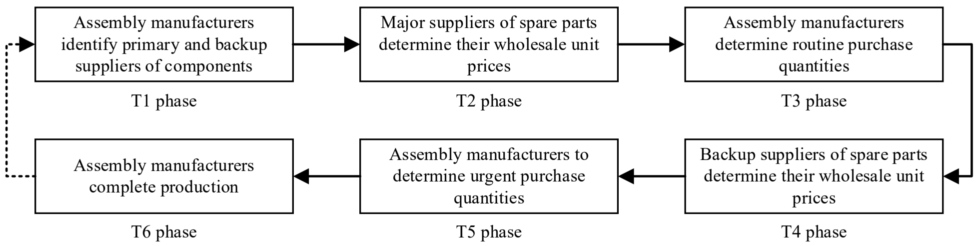

Fig. 2. Sequence of events for assembled supply chain decisions

3. Procurement decision analysis of assembly building supply chain under supply disruption risk

3.1 A dual-source procurement base model for assembly building supply chain under supply disruption risk

The study includes creating benchmark models to analyze the ideal order quantities for assembly manufacturers and the projected revenue functions of assembly supply chain members. This analysis is conducted in both centralized and decentralized decision-making scenarios. Meanwhile, we analyze whether the joint contract can enhance the supply capacity of the backup supplier of parts and components during supply disruptions, minimize the out-of-stock loss for the assembly manufacturer and downstream retailer, and achieve optimal coordination of the assembly supply chain. We use the change in revenue as a measure to assess the impact of different power structures on the assembly supply chain.

3.1.1 Assembly building supply chain operations under centralized decision-making

The aggregate profit of the assembly supply chain, taking into account the potential supply disruption from the primary component supplier, is:

$$\pi = p\min(q_i, x) + v\max(q_i - x, 0) - s\max(x - q_i, 0) - c_i q_i \tag{1}$$

The overall profitability of the assembly supply chain in the absence of any supply disruption risk from the primary component supplier is:

$$\pi = p\min(q_h + q_i, x) + v\max(q_h + q_i - x, 0) - s\max(x - q_h - q_i, 0) - c_h q_h - c_i q_i \tag{2}$$

At this point, the expected profit function of the assembly supply chain is:

$$\begin{aligned} E(\pi) &= \theta[p\min(q_i, x) + v\max(q_i - x, 0) - s\max(x - q_i, 0) - c_i q_i] + (1-\theta) \\ &[p\min(q_h + q_i, x) + v\max(q_h + q_i - x, 0) - s\max(x - q_h - q_i, 0) - c_h q_h - c_i q_i] \end{aligned} \tag{3}$$

The above equation can be rewritten as:

$$\begin{aligned} \pi &= \theta\int_0^{q_i}[px + v(q_i - x) - c_i q_i]f(x)dx + \theta\int_{q_i}^{\infty}[pq_i - s(x - q_i) - c_i q_i]f(x)dx + \\ &(1-\theta)\int_0^{q_h+q_i}[px + v(q_h + q_i - x) - c_h q_h - c_i q_i]f(x)dx + \\ &(1-\theta)\int_{q_h+q_i}^{\infty}[p(q_h + q_i) - s(x - q_h - q_i) - c_h q_h - c_i q_i]f(x)dx \end{aligned} \tag{4}$$

The first-order and second-order derivatives of Eq. (4) with respect to q_i can be obtained:

$$\begin{aligned} \frac{\partial \pi}{\partial q_i} &= (p - c_i + s) + (v - p - s)[\theta F(q_i) + (1-\theta)F(q_h + q_i)] \\ \frac{\partial^2 \pi}{\partial q_i^{\,2}} &= (v - p - s)[\theta f(q_i) + (1-\theta)f(q_h + q_i)] \end{aligned} \tag{5}$$

It is easy to get $\frac{\partial^2 \pi}{\partial q_i^{\,2}} < 0$, it can be seen that the formula (4) is a concave function on q_i, so that $\frac{\partial \pi}{\partial q_i} = 0$, to get the optimal supply of parts backup supplier should meet the conditions:

$$\theta F(q_i^{c*}) + (1-\theta)F(q_h + q_i^{c*}) = \frac{c_i - p - +s}{v - p - s} \tag{6}$$

Similarly, the second-order derivative of q_h is also less than 0. It can be seen that Eq. (4) is also a concave function about q_h, and there exists a maximum value, so that $\frac{\partial \pi}{\partial q_i} = 0, \frac{\partial \pi}{\partial q_h} = 0$, and get the optimal order quantity of the assembly supply chain under centralized conditions:

$$Q_C^* = q_h^* + q_i^{c*} = F^{-1}(\frac{c_h - p - s}{v - p - s}) \tag{7}$$

By analyzing Eqs. (7) and the $F(x)$ property, it can be concluded that the variation of the supply q_i^{c*} under centralized decision-making is closely related to the value of v. The higher the residual value of the unsold product held by the retailer, the more it motivates the backup supplier to maintain production of the product. By retaining a portion of the residual value, the backup supplier not only compensates for the purchasing losses of the assembly manufacturer, but also decreases their own production expenses. At the same time, the partial derivation of equation (5) with respect to θ is further obtained as $\frac{\partial^2 \pi}{\partial q_i \partial \theta} > 0$, and it is known that q_i varies in the same direction as θ, i.e., the larger the probability of supply disruption risk θ is, the larger the supply quantity of parts and components backup supplier is.

3.1.2 Assembly supply chain operations under decentralized decision-making

In the decentralized scenario, the model computation is simplified by assuming a constant supply amount q_h from the main parts supplier to the assembly producer. Concurrently, the conventional newsboy model is employed to determine the optimal quantity of parts provided by the backup supplier, and its supply capacity model is as follows:

$$L(q_i) = (1-\theta)E\min[(x-q_h), q_i] + \theta E\min(x, q_i)a = q_i - \theta\int_0^{q_i} F(x)dx - (1-\theta)\int_{q_h}^{q_h a + q_i} F(x)dx \tag{8}$$

The profit function, taking into account the production and delivery capacity limits of the parts backup provider, can be expressed as follows:

$$\begin{aligned}\pi_i(q_i) = {} & w_i L(q_i) + (1-\theta)[\int_{q_h}^{q_h+q_i} v(q_h+q_i-x)f(x)dx - \int_0^{q_h}(c_i - v)q_i f(x)dx \\ & -\int_{q_h}^{+\infty} c_i q_i f(x)dx] - \theta[\int_0^{q_i}[c_i - v(q_i - x)]f(x)dx + \int_{q_i}^{+\infty} c_i q_i f(x)dx]\end{aligned} \tag{9}$$

The first-order and second-order derivatives of Eqs. (9), respectively, are given by:

$$\frac{\partial \pi_i(q_i)}{\partial q_i} = w_i - c_i + [(1-\theta)F(q_h+q_i) + \theta F(q_i)](v - w_i) \tag{10}$$

$$\frac{\partial^2 \pi_i(q_i)}{\partial q_i^2} = [(1-\theta)f(q_h+q_i) + \theta f(q_i)](v - w_i) < 0 \tag{11}$$

It can be shown that the profit function $\pi_i(q_i)$ is a concave function with respect to q_i. There is a maximum value on the interval, which gives q_i^{d*} satisfying the following equation:

$$(1-\theta)F(q_h + q_i^{d*}) + \theta F(q_i^{d*}) = \frac{c_i - w_i}{v - w_i} \tag{12}$$

Then the profit function of a manufacturer with assembly is:

$$\begin{aligned}\pi_e(q_i) = {} & (1-\theta)[\int_0^{q_h+q_i}[w_e x + v(q_h+q_i-x)]f(x)dx + \int_{q_h+q_i}^{+\infty}[w_e(q_h+q_i) - \\ & s(x-q_h-q_i)]f(x)dx] + \theta[\int_0^{q_i}[w_e x + v(q_i - x)f(x)]dx + \\ & \int_{q_i}^{+\infty}[q_i w_e - s(x-q_i)]f(x)dx] - w_i L(q_i)\end{aligned} \tag{13}$$

Derivation of Eq. (12) gives:

$$\frac{\partial \pi_e(q_i)}{\partial q_i} = [\theta F(q_i) + (1-\theta)F(q_h + q_i)](w_i - w_e - s) + w_e - w_i \tag{14}$$

$$\frac{\partial^2 \pi_e(q_i)}{\partial q_i^{\,2}} = [\theta f(q_i) + (1-\theta)f(q_h + q_i)](w_i - w_e - s) \tag{15}$$

Making $\frac{\partial \pi_e(q_i)}{\partial q_i}$ equal to 0 yields the equation

$$\theta F(q_i) + (1-\theta)F(q_h + q_i) = \frac{w_i - w_e}{w_i - w_e - s} \tag{16}$$

And by the $F(x)$ characteristics of the formula (15) does not hold, $\frac{\partial \pi_e(q_i)}{\partial q_i} > 0$ constant, assembly manufacturer profit function $\pi_e(q_i)$ is a monotonically increasing function on the backup supplier of parts and components supply q_i, that is, the assembly manufacturer profit with the increase in the backup supply and increase. Under a decentralized choice, the retailer does not form any contractual agreement with the assembly maker. In this scenario, the retailer's anticipated profit can be determined as follows:

$$\begin{aligned} E(\pi_r) = \theta[p\min(q_i, x) + v\max(q_i - x, 0) - s\max(x - q_i, 0) - w_e q_i] + (1-\theta) \\ [p\min(q_h + q_i, x) + v\max(q_h + q_i - x, 0) - s\max(x - q_i - q_h, 0) - w_e(q_h + q_i)] \end{aligned} \tag{17}$$

That is, the above equation can be rewritten as:

$$\begin{aligned} \pi_r = \theta[\int_0^{q_i}[px + v(q_i - x) - w_e q_i]f(x)dx + \int_{q_i}^{\infty}[pq_i - s(x - q_i) - w_e q_i]f(x)dx] + \\ (1-\theta)[\int_0^{q_h+q_i}[px + v(q_h + q_i - x) - w_e(q_h + q_i)]f(x)dx + \\ \int_{q_h+q_i}^{\infty}[p(q_h + q_i) - s(x - q_h - q_i) - w_e(q_h + q_i)]f(x)dx] \end{aligned} \tag{18}$$

From Eq. (5) the same reason can be seen, the profit function π_r is about q_h , q_i concave function, so that $\frac{\partial \pi_r}{\partial q_i} = 0, \frac{\partial \pi_r}{\partial q_h} = 0,$ can get the retailer's optimal order quantity:

$$Q^* = q_h^{\,*} + q_i^{\,*} = F^{-1}(\frac{w_e - p - s}{v - p - s}) \tag{19}$$

After considering the analysis, it is evident that the decentralized decision-making model exposes both midstream assembly manufacturers and downstream retailers to the risk of supply disruptions. They must rely on a dual-source supply from the primary supplier of parts and components, the backup supplier of parts and components, or a single-source supply from the backup supplier of parts and components to meet the consumer market's demand. By contrasting Eq. (6) and Eq. (12), it is determined that the supply of spare parts backup suppliers is significantly greater under centralized decision-making than under decentralized decision-making. This is since the parts backup supplier, an independent operating entity, will opt to supply multiple assembly enterprises in order to maximize its own interests. This results in the dispersion of the supply quota to the assembly manufacturer, which in turn fails to optimize the overall revenue of the assembly supply chain under decentralized conditions. At the same time, the assembly manufacturer's expected profit is a monotonically increasing function of the back-up supply q_i, and expects the back-up supplier to be able to stabilize the supply by stocking a certain amount of parts in addition to the order production.

3.2 Assembly building supply chain under joint contracts

The assembly manufacturer will increase the number of orders to the backup supplier of parts and components to compensate for the loss caused by the shortage of goods in the event of a supply interruption, which can be easily caused by the existence of influencing factors such as a long transportation distance, a customs clearance process that is too time-consuming, and unqualified quality of goods. The upstream enterprises will sign repurchase contracts or revenue-sharing contracts with the downstream enterprises to reduce the out-of-stock loss caused by the supply disruption, improve the enthusiasm of the backup suppliers to supply, and maximize the profit of each participating entity in order to ensure the smooth operation of the supply chain.

3.2.1 Retailers' margins and purchase volumes under repurchase contracts

The assembly building supply chain operation risk is significantly higher than that of the typical supply chain. In order to optimize and coordinate the downstream supply chain, reduce oversupply retailer warehousing and sales costs, and return to the cash flow to alleviate the significant procurement concerns, the retailer will be assembled with the manufacturer's signed repurchase contract. At present, the retailer's profit function is as follows: there are still supply interruptions and no interruptions in two cases.

$$\begin{aligned}&E(\pi_{\mathrm{r}}{}') = \theta[p\min(q_i,x) + \gamma w_e \max(q_i - x,0) - s\max(x - q_i,0) - w_e q_i] + (1-\theta)\\&[p\min(q_h + q_i,x) + \gamma w_e \max(q_h + q_i - x,0) - s\max(x - q_i - q_h,0) - w_e(q_h + q_i)]\end{aligned} \tag{20}$$

Can be rewritten as:

$$\begin{aligned}&\pi_r{}^m = \theta[\int_0^{q_i}[px + \gamma w_e(q_i - x) - w_e q_i]f(x)dx + \int_{q_i}^{\infty}[pq_i - s(x - q_i) - w_e q_i]f(x)dx] +\\&(1-\theta)[\int_0^{q_h+q_i}[px + \gamma w_e(q_h + q_i - x) - w_e(q_h + q_i)]f(x)dx +\\&\int_{q_h+q_i}^{\infty}[p(q_h + q_i) - s(x - q_h - q_i) - w_e(q_h + q_i)]f(x)dx]\end{aligned} \tag{21}$$

The optimal order quantity of the retailer under the repurchase contract can be obtained by deriving q_h, q_i respectively: $Q^* = q_h{}^* + q_i{}^* = F^{-1}(\frac{w_e - p - s}{\gamma w_e - p - s})$

3.2.2 Profit and purchase volume of assembly manufacturers under joint contracts

At this point the assembly manufacturer's expected profit is:

$$\begin{aligned}&E(\pi_e{}^m) = \theta[w_e\min(q_i,x) - s\max(x - q_i,0) + (v - \gamma w_e)\max(q_i - x,0) - w_i q_i] + (1-\theta)\\&[w_e\min(q_h + q_i,x) - s\max(x - q_i - q_h,0) + (v - \gamma w_e)\max(q_h + q_i - x,0) - w_i q_i - w_h q_h]\end{aligned} \tag{22}$$

Rewrite the above equation in integral form as follows:

$$\begin{aligned}&\pi_e{}^m = \theta\int_0^{q_i}[w_e x + (v - \gamma w_e)(q_i - x) - w_i q_i]f(x)dx + \theta\int_{q_i}^{+\infty}[w_e q_i - s(x - q_i) - w_i q_i]\\&f(x)dx + (1-\theta)\int_0^{q_h+q_i}[w_e x + (v - \gamma w_e)(q_h + q_i - x) - w_i q_i - w_h q_h]f(x)dx +\\&(1-\theta)\int_{q_h+q_i}^{+\infty}[w_e(q_h + q_i) - s(x - q_h - q_i) - w_i q_i - w_h q_h]f(x)dx\end{aligned} \tag{23}$$

Similarly find the first and second order derivatives with respect to q_i

$$\frac{\partial \pi'}{\partial q_i} = (v - \gamma w_e - w_e - s)[(1-\theta)F(q_h + q_i) + \theta F(q_i)] + w_e + s - w_i \tag{24}$$

$$\frac{\partial^2 \pi'}{\partial q_i{}^2} = (v - \gamma w_e - w_e - s)[(1-\theta)f(q_h + q_i) + \theta f(q_i)] \tag{25}$$

From the assumptions, it is easy to know that $\frac{\partial^2 \pi'}{\partial q_i{}^2} < 0$ is always true, and we get the fulfillment condition of the optimal order quantity q_i^*:

$$(1-\theta)F(q_h + q_i^*) + \theta F(q_i^*) = \frac{w_i - w_e - s}{(v - \gamma w_e - w_e - s)} \tag{26}$$

3.2.3 Profitability of spare parts backup suppliers

Let the assembly manufacturer's expected profit from purchasing from the primary supplier of parts and components be π_{e1}, while the profit from purchasing from the backup supplier of parts and components be π_{e2}, and the expression of the corresponding expected profit function be:

$$\pi_{e1}(q_h) = (1-\theta)[\int_0^{q_h}[w_e x f(x)dx + \int_{q_h}^{+\infty}[w_e q_h f(x)dx + \int_0^{q_h} v(q_h - x)f(x)dx - \int_{q_h}^{q_h+q_1} s(x-q_h)f(x)dx] - \theta\int_{q_1}^{+\infty} sxf(x)dx \quad (27)$$

$$\pi_{e2}(q_i) = \pi_e{}^m(q_i) - \pi_{e1}(q_h) \quad (28)$$

There exists a revenue sharing factor κ where the assembly manufacturer shares the expected profit π_{e2} from the backup supplier to the backup supplier. The expected profit function of the backup supplier under the joint contract is obtained:

$$\pi_i{}^n = \pi_i + \kappa(\pi_e{}^m - \pi_{e1}) \quad (29)$$

This is obtained by bringing Eq. (9), Eq. (23), Eq. (27) into the above equation for q_i:

$$\frac{\partial \pi_i{}^n}{\partial q_i} = [(v-w_i) + \kappa(v - \gamma w_e - w_e - s)][(1-\theta)F(q_h+q_i) + \theta F(q_i)] + \kappa(w_e + s - w_i) + (w_i - c_i) \quad (30)$$

$$\frac{\partial^2 \pi_i{}^n}{\partial q_i{}^2} = [(v-w_i) + \kappa(v - \gamma w_e - w_e - s)][(1-\theta)f(q_h+q_i) + \theta f(q_i)] < 0 \quad (31)$$

That is, after joining the revenue sharing contract, there exists a maximum revenue for the backup provider and $\pi_i{}^n(q_i{}^{n*})$ satisfies the following equation:

$$(1-\theta)F(q_h + q_i{}^{n*}) + \theta F(q_i{}^{n*}) = \frac{c_i - w_i - \kappa(w_e + s - w_i)}{(v-w_i) + \kappa(v - \gamma w_e - w_e - s)} \quad (32)$$

Corollary 1: The gain-sharing coefficient κ should take values on the interval $[\frac{\pi_i(q_i{}^*) - \pi_i(q_i{}^{n*})}{\pi_e(q_i{}^{n*}) - \pi_{e1}(q_h)}, \frac{\pi_e(q_i{}^{n*}) - \pi_e(q_i{}^*)}{\pi_e(q_i{}^{n*}) - \pi_{e1}(q_h)}]$.

Proof: The assembly manufacturer enters a revenue-sharing contract with the backup supplier of parts and components in order to ensure that the contract serves as an ideal coordination mechanism. After the assembly manufacturer and the backup supplier join the contract, their respective revenues must not decrease from what they would have been under decentralized conditions.

$$\pi_e{}^n(q_i{}^{n*}) = \pi_e(q_i{}^{n*}) - \kappa[\pi_e(q_i{}^{n*}) - \pi_{e1}(q_h)] \ge \pi_e(q_i{}^*) \quad (33)$$

$$\pi_i{}^n(q_i{}^{n*}) = \pi_i(q_i{}^{n*}) + \kappa[\pi_e(q_i{}^{n*}) - \pi_{e1}(q_h)] \ge \pi_i(q_i{}^*) \quad (34)$$

Joining Eq. (33) and Eq. (34) yields a range of values for k:

$$\frac{\pi_i(q_i{}^*) - \pi_i(q_i{}^{n*})}{\pi_e(q_i{}^{n*}) - \pi_{e1}(q_h)} \le \kappa \le \frac{\pi_e(q_i{}^{n*}) - \pi_e(q_i{}^*)}{\pi_e(q_i{}^{n*}) - \pi_{e1}(q_h)} \quad (35)$$

When κ is within the above interval, the supply $q_i{}^{n*}$ of the backup supplier is closer to the supply $q_i{}^{c*}$ under the centralized decision than the supply $q_i{}^{d*}$ under the decentralized decision, and the expected profits of the backup supplier, the assembly manufacturer, and the retailer are larger than the expected profits of the relevant parties under the decentralized decision. It has been demonstrated that the assembly supply chain can be effectively coordinated in the event of a supply disruption crisis by establishing a joint contract that is based on buyback-revenue share. This approach achieves Pareto improvement in the supply chain.

4. Dual-source procurement decisions considering supplier capacity constraints under supply disruption risk

4.1 Purchasing decision for low-cost suppliers to be primary suppliers (NB Strategy)

In the NB strategy, the assembly manufacturer initially identifies that the low production cost component supplier B is the primary supplier, and the high production cost component supplier A is the backup supplier. The superscript " " is used to refer to the symbols of each parameter in the NB strategy, to distinguish them from the NA strategy. In order to resolve the Stackelberg game model that has been constructed through backward induction, it is necessary to analyze the emergency purchase order decision of the assembly manufacturer at stage T5, the wholesale pricing decision of the backup supplier of parts and components at stage T4, the assembly manufacturer's regular purchase order decision at stage T3, and the wholesale pricing decision of the main supplier of parts and components at stage T2. Setting the assembly manufacturer's order quantity of parts q_1^b in regular procurement, the probability of supply disruption risk of the primary supplier r_B . the wholesale unit price of primary supplier B is w_1^b , and the wholesale unit price of backup supplier A is w_2^b . The decision problem is to determine the quantity of the order q_2^b oriented to the backup supplier so as to maximize the revenue of the emergency procurement, i.e., the assembly manufacturer's profit in the T5 stage is:

$$\begin{aligned} &\pi_M^b = \max\ [q_2^b[D - q_2^b - q_1^b(1-\mathrm{r}_B)] - q_2^b w_2^b] \\ &s.t. \quad 0 \le q_2^b \le \varphi_A \end{aligned} \tag{36}$$

The decision problem at this point is to find the optimal wholesale unit price w_2^b for the emergency purchase that maximizes the profit of the backup supplier A at T4:

$$\pi_A^b = \max\ [(\mathrm{w}_2^b - \mathrm{c}) q_2^b] \tag{37}$$

Assume that the wholesale unit price w_1^b of the main supplier B is known, the decision problem is to determine the optimal order quantity q_1^b for routine purchases, and the maximum expected profit of the assembly manufacturer in stage T3 is:

$$\begin{aligned} &E(\pi_M^b) = \max\ [E(\mathrm{r}_B)[q_2^b + q_1^b(1-\mathrm{r}_B)][D - q_2^b - q_1^b(1-\mathrm{r}_B)] - q_2^b w_2^b - q_1^b w_1^b(1-\mathrm{r}_B)] \\ &s.t. \quad 0 \le q_1^b \le \varphi_B \end{aligned} \tag{38}$$

The decision problem of major supplier B in stage T2 is to determine the optimal wholesale unit price w_1^b in regular purchasing and to obtain its own maximum expected profit function as follows:

$$\mathrm{E}(\pi_B^b) = \max\ [\mathrm{E}(\mathrm{r}_B)(w_1^b - c_B)(1-\mathrm{r}_B) q_1^b] \tag{39}$$

As illustrated in Theorem 1, the wholesale pricing decisions of various parts suppliers and the optimal purchasing decisions of assembly manufacturers in the NB strategy are realized by resolving the four objective planning functions.

Theorem 1: In the NB strategy, the optimal wholesale unit price of the main supplier of parts B under regular purchasing is:

$$w_1^{b*} = \max\left\{(5D + 3c + 8\mathrm{c}_B)/16, 11\varphi_B(h^2 - 3h + 3)/(h-2)12 - (5D + 3c)/8\right\} \tag{40}$$

The optimal order quantity for an assembly manufacturer to make a routine purchase is:

$$q_1^{b*} = \min[3(2-h)(5D + 3c - 8c_B)/44(h^2 - 3h + 3), \varphi_B] \tag{41}$$

The optimal wholesale unit price for spare parts backup supplier A under emergency purchasing is:

$$w_2^{b*} = [\mathrm{D+c} - q_1^{b*}(1-\mathrm{r}_B)]/2 \tag{42}$$

The optimal order quantity for an assembly manufacturer to make an emergency purchase is:

$$q_2^{b*} = [D - c - q_1^{b*}(1-\mathrm{r}_B)]/4 \tag{43}$$

Theorem 1 shows that when the low-cost component supplier B is chosen as the main supplier, under its capacity constraints, the assembly manufacturer's regular purchase quantity decision q_1^{b*} is mainly affected by the potential market demand D, the production cost of emergency purchases c, the wholesale price per unit of the main supplier w_1^b, and the reliability coefficient of the supplier h (which indicates the level of probability of the risk of disruption). Where $\partial q_1^b / \partial w_1^b < 0$, $\partial q_1^b / \partial D > 0$, $\partial q_1^b / \partial c > 0$, as the wholesale unit price of the conventional procurement channel increases, the assembly manufacturer's conventional procurement quantity of parts and components will be reduced, and vice versa, with the potential market demand, the urgent procurement cost will be prompted to increase the assembly manufacturer's conventional procurement quantity. This suggests that although the main supplier of parts and components currently is the lower-cost supplier B, there is still a competing interest with the backup supplier A.

When Supplier A, a higher-cost component supplier, is selected as a backup supplier, under its capacity constraints, the assembly manufacturer's emergency purchase quantity decision q_2^{b*} is mainly affected by the potential market demand D, the production cost of the emergency purchase c and the quantity $q_1^{b*}(1-\mathrm{r}_B)$ supplied by primary supplier B when it is routinely purchased by Supplier B. That is, the quantities supplied by a major supplier in the case of regular purchases are determined by the production capacity and the probability of output under the risk of supply disruptions. This implies that emergency purchases not only compensate for shortfalls in the production capacity of a major supplier of parts and components, but also effectively respond to the risk of supply disruptions.

$\partial w_1^b / \partial c > 0$, $\partial w_2^b / \partial c > 0$. This implies that, even though routine purchases occur prior to emergency purchases, the production costs of emergency purchases have an impact on both primary and secondary suppliers. The wholesale unit price of the primary supplier decreases as the cost of emergency production decreases, and the reverse is also true.

According to Eq. (39) and Eq. (40), the expected profit of the main supplier B of parts and components under NB strategy is obtained as:

$$\mathrm{E}(\pi_B^b) = \frac{h(q_1^{b*})(5D+3c-8\mathrm{c}_B)}{32} - \frac{11}{24}h^2(q_1^{b*})^2 \tag{44}$$

According to Eq. (37), Eq. (42), and Eq. (43), the expected profit of parts backup supplier A under NB strategy is obtained as:

$$E(\pi_A^b) = \frac{h^2(q_1^{b*})^2}{24} - \frac{h(q_1^{b*})(D-c)}{8} + \frac{(D-c)^2}{8} \tag{45}$$

Similarly, according to Eq. (38) and Theorem 1, the expected profit of the assembly manufacturer under the NB strategy is obtained as:

$$E(\pi_M^b) = \frac{(D-c)^2}{16} + \frac{11}{48}h^2(q_1^{b*})^2 \tag{46}$$

Then the expected profit of the assembly supply chain as a whole is:

$$E(\pi_S^b) = \frac{h(q_1^{b*})\left(D+7c-8\mathrm{c}_B\right)}{32} - \frac{3h^2(q_1^{b*})^2}{16} + \frac{3(D-c)^2}{16} \tag{47}$$

4.2 Purchasing decision for high-cost suppliers to be primary suppliers (NA Strategy)

In the NA strategy, the assembly manufacturer initially determines that component supplier A with higher production costs is the primary supplier and component supplier B with lower production costs is the backup supplier. The superscript " " is used to refer to the symbols of each parameter under the NA strategy to differentiate it from the NB strategy. The analytical solution process is briefly summarized below since the mathematical planning model established here is analogous to the one above. Assuming that the assembly manufacturer's parts order quantity q_1^a in routine purchasing is known, the supply disruption risk probability r_A of the primary supplier, the wholesale unit price w_1^a of the primary supplier A, and the wholesale unit price w_2^a of the backup supplier B. The assembly manufacturer's purchasing decision problem in stage T5 is to ensure that the emergency purchases will be sufficiently profitable.

$$\pi_M^a = \max \ [q_2^a[D - q_2^a - q_1^a(1-\mathrm{r}_A)] - q_2^a w_2^a] \quad s.t. \quad 0 \le q_2^a \le \varphi_B \tag{48}$$

Assume that the assembly manufacturer's order quantity of parts in the regular procurement is known q_1^a, the supply disruption risk probability r_A of the main supplier, and the wholesale unit price w_1^a of the main supplier A. The decision problem at this point is to find the optimal wholesale unit price w_2^a for the emergency procurement to maximize the profitability of the back-up supplier B in stage T4.

$$\pi_B^a = \max \ [(w_2^a - \mathrm{c})q_2^a] \tag{49}$$

Assuming that the wholesale unit price w_1^a of the main supplier of components A is known, the assembly manufacturer's decision problem in stage T3 is to find the optimal order quantity q_1^a for routine purchases to maximize the expected profit.

$$E(\pi_M^a) = \max \ [E(\mathrm{r}_A)[q_2^a + q_1^a(1-\mathrm{r}_A)][D - q_2^a - q_1^a(1-\mathrm{r}_A)] - q_2^a w_2^a - q_1^a w_1^a(1-\mathrm{r}_A)] \quad s.t. \quad 0 \le q_1^a \le \varphi_A \tag{50}$$

The decision problem of major supplier A in stage T2 is to determine the optimal wholesale unit price w_1^a in regular purchasing to maximize the expected profit is maximized.

$$\mathrm{E}(\pi_A^a) = \max \ [\mathrm{E}(\mathrm{r}_A)(w_1^a - c_A)(1-\mathrm{r}_A)q_1^a] \tag{51}$$

Similarly, solving the above four objective planning functions yields the wholesale pricing decisions of different parts suppliers and the optimal purchasing decisions of assembly manufacturers in the NA strategy, as shown in Theorem 2.

Theorem 2: In the NA strategy, the optimal wholesale unit price of the main supplier A of parts under regular purchasing is:

$$w_1^{a*} = \max\left\{(5D + 3c + 8\mathrm{c}_A)/16, 11\varphi_A(h^2 - 3h + 3)/(h-2)12 - (5D + 3c)/8\right\} \tag{52}$$

The optimal order quantity for an assembly manufacturer to make a routine purchase is:

$$q_1^{a*} = \min[3(2-h)(5D + 3c - 8c_A)/44(h^2 - 3h + 3), \varphi_A] \tag{53}$$

The optimal wholesale unit price for spare parts backup supplier B under emergency purchasing is:

$$w_2^{a*} = [\mathrm{D} + \mathrm{c} - q_1^{a*}(1-\mathrm{r}_A)]/2 \tag{54}$$

The optimal order quantity for an assembly manufacturer to make an emergency purchase is:

$$q_2^{a*} = [D - c - q_1^{a*}(1-\mathrm{r}_A)]/4 \tag{55}$$

And similarly, the expected profit expression of each subject of the assembly supply chain under NA strategy can be obtained according to Theorem 2.

The expected profit of the main supplier of components A now is:

$$\mathrm{E}(\pi_A^a) = \frac{h(q_1^{a*})(5D + 3c - 8c_A)}{32} - \frac{11}{24}h^2(q_1^{a*})^2 \tag{56}$$

The expected profit of Parts Backup Supplier B is:

$$E(\pi_B^a) = \frac{h^2(q_1^{a*})^2}{24} - \frac{h(q_1^{a*})(D-c)}{8} + \frac{(D-c)^2}{8} \tag{57}$$

Similarly, the expected profit of the assembly manufacturer under the NA strategy is obtained as:

$$E(\pi_M^a) = \frac{(D-c)^2}{16} + \frac{11}{48} h^2 (q_1^{a*})^2 \tag{58}$$

In summary, the overall expected profit of the assembly building supply chain is:

$$E(\pi_S^a) = \frac{h(q_1^{a*})\left(D+7c-8c_A\right)}{32} - \frac{3h^2(q_1^{a*})^2}{16} + \frac{3(D-c)^2}{16} \tag{59}$$

4.3 Comparison of two vendor allocation strategies

The comparison of the wholesale unit prices of the primary and fallback suppliers of parts and components can be used to derive Corollary 2.

Corollary 2: The expected wholesale unit price of the parts backup supplier will always be significantly higher than the expected wholesale unit price of the primary parts supplier, regardless of whether the assembly manufacturer selects the NB or NA sourcing strategy. Specifically, there are $E(w_2^{b*}) > w_1^{b*}$, $E(w_2^{a*}) > w_1^{a*}$ in this scenario.

According to Corollary 2, the primary supplier of parts consistently maintains a wholesale price advantage over the backup supplier of parts, irrespective of the assembly manufacturer's selection of either NB or NA strategy. This is due to the fact that the primary supplier is prepared to reduce the wholesale price and forgo a portion of its revenue in the event of a supply disruption, in order to preserve a long-term cooperative relationship with the assembly manufacturer.

Corollary 3 can be obtained by comparing the wholesale unit price decisions of major suppliers of components under NB and NA strategies.

Corollary 3: If there is no capacity constraint limiting $\varphi_B = \varphi_A$, there are $0 \le w_1^{a*} - w_1^{b*} \le (c_A - c_B)/2$ and $\partial(w_1^{a*} - w_1^{b*})/(\partial h) \ge 0$.

In the absence of capacity constraints, the wholesale unit price of a low production cost supplier as a major supplier is still lower than the wholesale unit price of a high production cost supplier, $w_1^{b*} \le w_1^{a*}$. However, the difference in production costs between the main suppliers of components under the NB and NA strategies will be higher than the difference in wholesale unit prices between them, i.e., $\partial(w_1^{a*} - w_1^{b*})/(\partial h) \ge 0$. At the same time, $\partial(w_1^{a*} - w_1^{b*})/(\partial h) \ge 0$ also shows that the aggressiveness of major suppliers of parts and components regarding wholesale asking prices increases with supply reliability.

Corollary 4: For the assembly manufacturer, if there exists a threshold k_1, the NA strategy will outperform the NB strategy when $\varphi_B < k_1$, i.e., $E(\pi_M^a) > E(\pi_M^b)$. If $\varphi_B \ge k_1$, there is an NB strategy that outperforms the NA strategy, i.e., $E(\pi_M^b) \ge E(\pi_M^a)$, where the threshold $k_1 = q_1^{a*}$.

When $k_1 \le \varphi_B < \varphi_A$, although the capacity of component supplier A is higher than that of supplier B, the difference is not significant, and the capacity advantage of A is not obvious, and it is still dominated by B's production cost advantage. Therefore, at this time, the assembly manufacturer will likewise choose the NB strategy. Therefore, at this point the assembly manufacturer will likewise choose the NB strategy. When $\varphi_B < k_1$, the capacity of component supplier B is small, and in comparison, the capacity advantage of supplier A dominates, at which point it is better for the assembly manufacturer to have A as the main supplier.

Corollary 5: As the probability of supplier supply disruption risk decreases and supply reliability increases, the predominance interval of the NB strategy increases and the predominance interval of the NA strategy decreases for the assembly manufacturer, i.e., $\partial k_1 / \partial h \le 0$. Meanwhile, as the production cost of emergency purchases by the backup supplier increases, the dominance interval of the NA strategy increases while the dominance interval of the NB strategy decreases, i.e., $\partial k_1 / \partial c \ge 0$.

From Corollary 5, as the supply reliability of the primary supplier of parts in routine production increases, the threshold k_1 in Fig. 3 moves to the left, and the strategy interval in which the low-cost supplier is the dominant primary supplier becomes larger. And as the production cost of the backup supplier of parts increases in emergency procurement, the threshold k_1 will move to the right, i.e., the strategy interval in which the high-cost supplier is the dominant primary supplier will become larger.

Corollary 6: For the assembly supply chain as a whole, if there exists a threshold k_2, when $\varphi_B < k_2$, there is a NA strategy that outperforms the NB strategy, i.e., $E(\pi_S^a) > E(\pi_S^b)$. When $\varphi_B \geq k_2$, there is a NB strategy that outperforms the NA strategy, i.e., $E(\pi_S^b) \geq E(\pi_S^a)$, where the threshold k_2 is: $k_2 = \frac{D + 7c - 8c_B + [(12hq_1^{a*} + 8c_B - 7c - D)^2 + 192q_1^{a*}h(c_A - c_B)]^{1/2}}{12h}$.

From Corollary 6, if the capacity of low-cost supplier B is high $(\varphi_B \geq k_2)$, choosing the NB strategy is the optimal supplier allocation scheme, and conversely, if the capacity of low-cost supplier B is low $(\varphi_B < k_2)$, the NA strategy needs to be chosen in order to induce the supply chain as a whole to realize higher expected returns. When $k_2 < \varphi_A < \varphi_B$, component supplier B has both high capacity and low production cost advantages and chooses the NB strategy as optimal. When $k_2 < \varphi_B < \varphi_A$, parts supplier B has low production costs, parts supplier A has higher capacity, but the capacity of the two is close, so the capacity advantage of supplier A is not superior to the cost advantage of supplier B, still choose the NB strategy optimal. When $\varphi_B < k_2$, φ_A is much larger than φ_B. The capacity advantage of the component suppliers will play a dominant role, thus choosing the NA strategy is more favorable to the assembly supply chain.

Corollary 7: There is some deviation (distortion) in the selection of the optimal strategy for dual-source procurement corresponding to $k_1 > k_2$ for the assembly manufacturer with the entire supply chain system. Also the range of the intervals of the deviations of the two strategies $(k_1 - k_2)$ is monotonically increasing related to the supply reliability h of the main supplier, i.e. $\partial(k_1 - k_2)/\partial h > 0$. And is monotonically decreasingly related to the emergency production cost c of the backup supplier, $\partial(k_1 - k_2)/\partial c < 0$.

According to Corollary 7, there is a bias between the assembly manufacturer and the assembly supply chain in the decision making of component supplier allocation strategy. For the assembly manufacturer, and the assembly supply chain as a whole, respectively, if there is $\varphi_B > k_1$, then both believe that the NB strategy will prevail. If there is $\varphi_B < k_2$, both believe that NA strategy will be superior. And when $k_2 \leq \varphi_B < k_1$, the assembly manufacturer tends to think that NA strategy is superior and the assembly supply chain as a whole tends to think that NB strategy is superior. That is, there is a bias between the assembly manufacturer and the assembly supply chain in the selection of high and low-cost suppliers as the main suppliers.

Also from Corollary 7, the deviation interval between the assembly manufacturer's and the assembly supply chain supplier's optimal allocation strategy becomes larger as the production cost of the backup supplier in the emergency sourcing phase decreases $(\partial(k_1 - k_2)/\partial c < 0)$ .This is due to the fact that the NA strategy is currently dominant, and the high-cost parts supplier A has a substantial high-capacity advantage. However, the high-capacity advantage of supplier A will be weakened as a result of the reduction in procurement and production costs in the emergency phase, which will result in a decrease in the number of parts purchased in the routine phase. At this juncture, the predominance zones for both NA strategies are decreasing, while the predominance zones for both NB strategies are increasing for the assembly manufacturer and the assembly building supply chain as a whole. The deviation interval of the supplier's optimal allocation strategy decreases $(\partial(k_1 - k_2)/\partial h > 0)$ as the probability of supply disruption risk increases, i.e., supply reliability diminishes.

5. Simulation analysis

5.1 Sensitivity analysis of base model parameters

In a three-tier assembly supply chain consisting of a primary supplier of parts and components, a backup supplier, an assembly manufacturer, and a retailer, the values of the corresponding parameters are set as follows, taking into account actual and hypothetical conditions $c_i = 10$, $c_h = 8$, $w_i = 20$, $w_h = 15$, $w_e = 25$, $p = 40$, $s = 20$, $v = 5$, where the uncertain market demand follows a normal distribution of $N(100, 10^2)$.

In order to determine the expected profit of the parts backup supplier, assembly manufacturer, and the entire assembly supply chain under the three scenarios of centralized decision making, decentralized decision making, and joint contracting, assuming $\gamma = 0.91$ and $\kappa = 0.3$. The probability of supply disruption risk is subject to variation within the interval [0, 0.9]. These results are illustrated in Fig. 3, Fig. 4, and Fig. 5. It is evident that the expected profit of the assembly manufacturer decreases as the probability of supply disruption risk increases, whereas the expected profit of the backup supplier increases. It is also found that the expected profits of assembly manufacturers and parts backup suppliers will be significantly higher than the decentralized decision-making model after the signing of a joint contract. As the probability of supply disruption risk increases, the gap between the expected returns of assembly manufacturers and the decentralized decision-making model becomes wider, while the gap between the expected returns of parts backup suppliers and the decentralized decision-making model gets narrower.

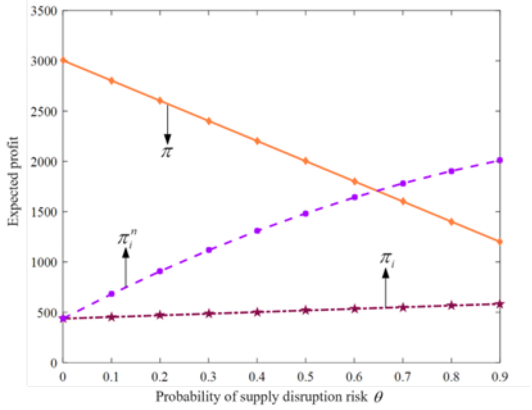

Fig. 3. Impact of supply disruption risk probability θ on π, π_i and π_i^n

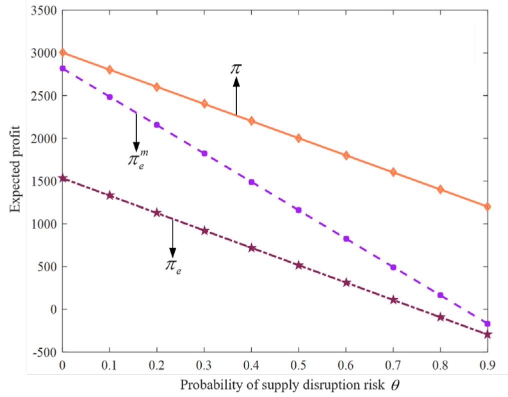

Fig. 4. Impact of supply disruption risk probability θ on π, π_e and π_e^m

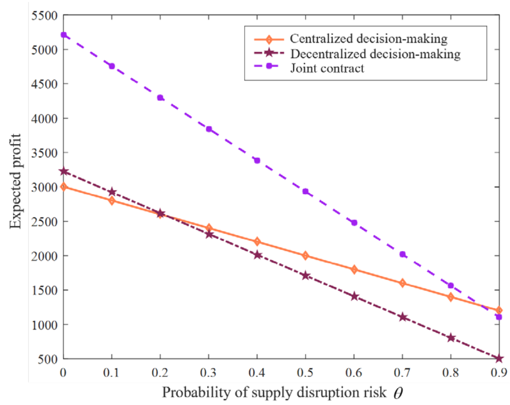

Fig. 5. Impact of θ on the overall expected profit of the assembly supply chain under different decision models

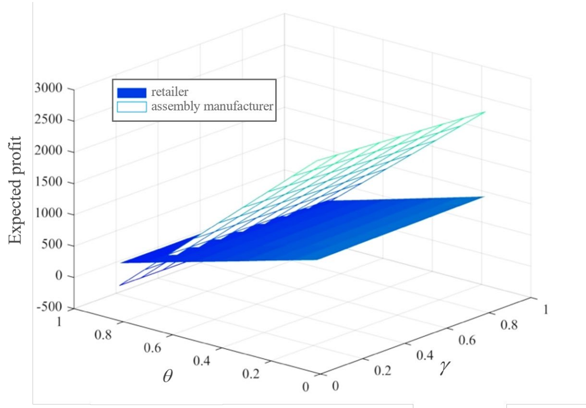

Fig. 6. Effect of parameters θ, γ on the expected profit of assembly manufacturers, retailers

Given $q_i = 60$, $q_h = 50$, the influence of several parameter combinations (γ, κ, θ) on the optimization effect of assembly supply chain joint contract coordination is illustrated in Fig. 6 and Fig. 7. Compared to the revenue sharing coefficient, the repurchase coefficient has a minimal impact on the expected profit of assembly manufacturers and retailers. As the repurchase coefficient increases, the expected profit of assembly manufacturers will slightly decrease, while the expected profit of retailers will slightly increase. Furthermore, Fig. 7 illustrates a clear correlation between the rise in the revenue sharing ratio and the substantial increase in the predicted revenue of the parts backup provider.

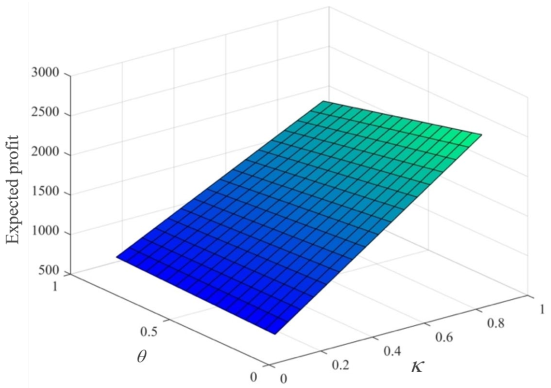

Fig. 7. Effect of parameters κ, θ on the expected profit of parts backup suppliers

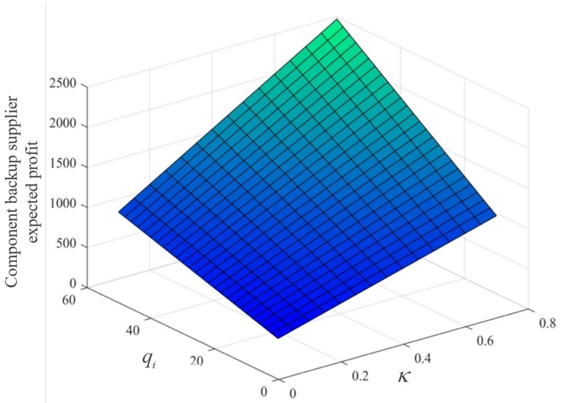

Fig. 8. Effect of revenue sharing factor on procurement volume and expected profit of parts backup suppliers

Assuming $\theta = 0.3$, the variation of expected profit and supply volume of parts backup supplier in the range of revenue sharing coefficient [0, 0.9] interval is shown in Fig. 8. Increasing the revenue sharing coefficient has been observed to boost the expected profit of the parts backup supplier and the supply volume, which refers to the purchasing volume of the assembly manufacturer. This finding further confirms that the joint contract enables optimal coordination of the assembly supply chain and the presence of an optimal ordering strategy. Consequently, the assembly supply chain can achieve or surpass the level of profitability achieved under centralized decision making.

5.2 Analysis of purchasing decisions of low-cost suppliers of parts and components

In the regular sourcing stage, supplier B has a low unit cost of production $c_B = 3$ and supplier A has a high unit cost of production $c_A = 4$. In the emergency sourcing stage, both suppliers A and B have a unit production cost of $c = 5$, and the high-cost supplier A has a production capacity of $\varphi_A = 75$. The potential market demand is $D = 110$. The assembly manufacturer decides to choose NB strategy or NA strategy according to the expected profit it obtains. Substituting the parameters into Eqs. (46) and (58), it can be seen that the values of $E(\pi_M^b)$ and $E(\pi_M^a)$ are related to φ_A and h. Taking $h = 0.7$ and $h = 0.6$, respectively, we obtain the trend of $E(\pi_M^b)$ and $E(\pi_M^a)$ in the interval $\varphi_A \in [50,110]$, as shown in Fig. 9.

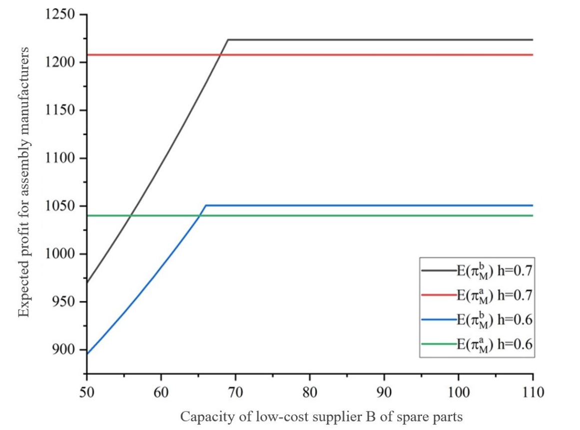

Fig. 9. Impact of low-cost supplier capacity, reliability on assembly manufacturer's expected profit

As can be seen in Fig. 9, the value of the assembly manufacturer's expected profit, $E(\pi_M^a)$, is not correlated with the production capacity, φ_B, of the low-cost supplier of parts, B, under the NA strategy, which are both horizontal straight lines. When the supply reliability $h = 0.7$, the curve $E(\pi_M^b)$ changes from a monotonically increasing function to a fixed value at the inflection point $\varphi_B = 69$. This is because the quantity of parts routinely purchased under the NB strategy is affected by a combination of Supplier B's production capacity and the probability of supply disruption risk, and the quantity of routinely purchased is determined when the supply reliability is fixed, at which point $E(\pi_M^b)$ will only vary with φ_B. $E(\pi_M^a)$ and $E(\pi_M^b)$ intersect at point $\varphi_B = 67$, i.e., the low-cost supplier of parts and components is better off executing the NA strategy when its B-capacity is lower than 67, and is better off executing the NB strategy when it is higher than 67. It can further be seen that as supply reliability increases and the probability of supply disruption risk decreases, the point of intersection of the $E(\pi_M^a)$ and $E(\pi_M^b)$ curves shifts to the left (the low-cost supplier's B-capacity threshold decreases), and the assembly manufacturer's expected profit increases, while the decision bias between the two supplier allocation strategies becomes larger.

6. Conclusion

This study examines the organized coordination of joint contracting in the assembly supply chain, taking into account the potential risk of supply disruption. It does so by developing a dual-source procurement model that incorporates various decision-making modes of power. This model introduces a buyback contract between the retailer and the assembly manufacturer, as well as a revenue-sharing contract between the parts backup supplier and the assembly manufacturer. The study demonstrates that implementing a joint contract can incentivize backup suppliers of parts and components, enhance the stockpile of components, fulfill market demand in the event of supply disruptions from primary suppliers, mitigate supply disruption crises, and ultimately optimize and coordinate the entire supply chain. Additionally, the joint contract can enhance the profitability of retailers and backup suppliers of parts and components by fairly redistributing income. Consequently, assembly manufacturers can effectively mitigate the negative effects of supply disruptions by implementing a combination of diverse contractual agreements, strengthening the ability to acquire from several sources, and establishing stronger partnerships with backup suppliers and retailers.

Additionally factoring in the potential for supply disruptions and limitations in the capacity of component suppliers, it may not be an ideal approach for the assembly building supply chain if the assembly manufacturer designates the primary component supplier as a provider with low production costs. Assembly manufacturers prioritize high-cost suppliers when facing capacity constraints, despite the cost advantages of low-cost suppliers. They only turn to low-cost suppliers as their main suppliers when these suppliers have sufficient capacity or when the gap between them and competing firms is not substantial. When a supplier that offers parts and components at a lower cost is the primary supplier, the wholesale unit price it charges the assembly manufacturer is more competitive compared to a supplier that offers parts and components at a higher cost. As a result, the assembly manufacturer tends to choose the higher-cost supplier as its primary supplier in its purchasing decision (NA strategy). In the context of the assembly building supply chain, when the capacity of low-cost component suppliers is limited, it is more beneficial to choose high-cost suppliers as the main supplier in order to maximize the overall expected profit. Conversely, when the capacity of low-cost suppliers is larger, selecting them as the main supplier strategy leads to greater overall expected profit. This paper presents a different finding compared to previous studies. It shows that when the cost of emergency sourcing is low, it widens the gap between the assembly manufacturer's best sourcing strategy and the overall best returns of the assembly supply chain. On the other hand, when the risk of supply disruption is high, it reduces the gap between the assembly manufacturer's best sourcing strategy and the overall best returns of the assembly supply chain.

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Data Availability Statement

The data obtained from the simulation experiments, which support the conclusions of this study, are provided in the appropriate portions of the text.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the pub-lication of this paper.

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Appendix

Theorem 1 Proof:

The second-order derivative of Eq. (36) above with respect to the number of orders q_2^b shows that, $\frac{d^2\pi_M^b}{d(q_2^b)^2}<0$, i.e., π_M^b is a concave function with respect to q_2^b, so that the first-order condition is equal to 0 and $q_2^b = D-w_2^b+q_1^b(\mathrm{r}_B-1)/2$ is obtained. Combined with the limiting conditions of the backup supplier's capacity constraints, the optimal solution of the plan is obtained:

$$q_2^b = \max\left\{0,\min[D-w_2^b+q_1^b(\mathrm{r}_B-1)/2,\varphi_A]\right\}$$

Bringing Eq. (40) into Eq. (37) yields the expression for the segmented function of $\pi_A^b(w_2^b)$:

$$\pi_A^b(w_2^b)=\begin{cases}(w_2^b-\mathrm{c})[D-w_2^b+q_1^b(\mathrm{r}_B-1)/2], & D-q_1^b(\mathrm{r}_B-1)-2\varphi_A\le w_2^b\le D-q_1^b(\mathrm{r}_B-1)\\(w_2^b-\mathrm{c})\varphi_A, & w_2^b\le D-q_1^b(\mathrm{r}_B-1)-2\varphi_A\\0, & w_2^b\ge D-q_1^b(\mathrm{r}_B-1)\end{cases}$$

It is clear that the above formula is a continuous function on w_2^b, to solve the first segment of the function of the second-order derivatives on w_2^b,it can be seen that it is less than 0, and the first-order conditional solution for $\mathrm{D+c}+q_1^b(\mathrm{r}_B-1)/2$, the second segment of the function of the monotonically increasing function on w_2^b. Also combined with assumption 4 it can be seen to exist: $D-q_1^b(\mathrm{r}_B-1)-2\varphi_A \le \mathrm{D+c+}q_1^b(\mathrm{r}_B-1)/2 \le D-q_1^b(\mathrm{r}_B-1)$, That is, the above segmented function as a whole is a single peaked function and there exists a maximum point whose expression is shown in (41).

$$w_2^b = [\mathrm{D+c+}q_1^b\left(\mathrm{r}_B-1\right)]/2$$

Substituting Eq. (40) and Eq. (41) into Eq. (38) and simplifying gives:

$$E(\pi_M^b) = \int_0^h \left[[q_2^b + q_1^b(1-\mathrm{r}_B)][D-q_2^b-q_1^b(1-\mathrm{r}_B)] - q_2^b w_2^b - q_1^b w_1^b (1-\mathrm{r}_B) \right] \frac{1}{h} dq_1^b$$
$$= q_1^b w_1^b (\frac{h}{2}-1) + \frac{(D-c)^2 + q_1^b(2-h)(5D+3c)}{16} + \frac{11(3h-3-h^2)(q_1^b)^2}{48}$$

A partial derivative of the above equation easily shows that the function $E[\pi_M^b(q_1^b)]$ is a concave function with respect to q_1^b, which gives $q_1^b = 3(2-h)(5D+3c-8w_1^b)/22(h^2-3h+3)$ by making its first-order derivative equal to zero. Meanwhile, combining with the limiting conditions of the capacity constraints of the main suppliers, we obtain the optimal solution of the plan:

$$q_1^b = \max\left\{0, \min[3(2-h)(5D+3c-8w_1^b)/22(h^2-3h+3), \varphi_B]\right\}$$

Substituting Eq. (42) into Eq. (39) yields the following expression for the segmented function:

$$\mathrm{E}(\pi_B^b) = \begin{cases} \dfrac{3h(w_1^b-c_B)(h-2)^2(5D+3c-8w_1^b)}{44(h^2-3h+3)}, \dfrac{11\varphi_B(h^2-3h+3)}{(h-2)12} - \dfrac{5D+3c}{8} < w_1^b < \dfrac{5D+3c}{8} \\ \dfrac{(w_1^b-c_B)h\varphi_B}{2}, \ w_1^b \le \dfrac{11\varphi_B(h^2-3h+3)}{(h-2)12} - \dfrac{5D+3c}{8} \\ 0, \ w_1^b \ge \dfrac{5D+3c}{8} \end{cases}$$

Obviously, the above formula is a continuous function on w_1^b, solve the second-order partial derivative of the first segment function on w_1^b, it can be seen that it is less than 0 for the concave function, and then get the first-order conditional solution for the $\left(5D+3c+c_B\right)/16$, the second segment function is a monotonically increasing function on w_1^b, so the above segment function as a whole for the right-hand side of the coordinates of the single-peaked function. The point of maximum value of this segmented function is obtained:

$$w_1^b = \max\left\{(5D+3c+\mathrm{c}_B)/16, 11\varphi_B(h^2-3h+3)/(h-2)12-(5D+3c)/8\right\}$$

Corollary 2 Proof:

Let $G_B = 3(2-h)(5D+3c-8c_B)/44(h^2-3h+3)$, and can be obtained by substituting Eq. (41) into Eq. (42) and solving for the expectation:

$$E(w_2^{b*}) = \begin{cases} \dfrac{15D+9c-24\mathrm{cb}}{176} + \dfrac{h\left(73D+79c+24\mathrm{cb}\right)}{176} + \dfrac{3\left(2h-3\right)\left(5D+3c-8\mathrm{cb}\right)}{176\left(h^2-3h+3\right)}, \varphi_B > G_B \\ \dfrac{\varphi_B h^2}{4} + \dfrac{h\left(D-\varphi_B+c\right)}{2}, \varphi_B \le G_B \end{cases}$$

According to Eq. (40) there is:

$$E(w_2^{b*})-w_1^{b*}=\begin{cases}\dfrac{h(73D+79c+24c_B)}{176}+\dfrac{3(2h-3)(5D+3c-8c_B)}{176(h^2-3h+3)}-\dfrac{(3c+14c_B+5D)}{22},\varphi_B>G_B\\ \dfrac{5D+3c+2\varphi_B h^2+4h(D-\varphi_B+c)}{8}-\dfrac{11\varphi_B(h^2-3h+3)}{12(h-2)},\varphi_B\le G_B\end{cases}$$

According to the assumption conditions, it is obvious that there is $h(73D+79c+24c_B)/176>(3c+14c_B+5D)/22$, the first function of the above equation is greater than 0. $11\varphi_B(h^2-3h+3)/12(h-2)<0$, the second function of the above equation is also greater than 0, and we get $E(w_2^{b*})-w_1^{b*}>0$, i.e., $E(w_2^{b*})>w_1^{b*}$. Similarly, there is $E(w_2^{a*})>w_1^{a*}$, and the proof is complete.

Corollary 3 Proof:

Let $\varphi_B=\varphi_A=\varphi$, $G_A=3(2-h)(5D+3c-8c_A)/44(h^2-3h+3)$, obtained according to Equations (40) and (52):

$$w_1^{a*}-w_1^{b*}=\begin{cases}\dfrac{c_A-c_B}{2},\varphi>G_B\\ \dfrac{15D+9c+8c_A}{16}-\dfrac{11\varphi(h^2-3h+3)}{12(h-2)},G_A<\varphi\le G_B\\ 0,\varphi\le G_A\end{cases}$$

In the second function of the above equation, since $G_A<\varphi\le G_B$, it is easy to see that $0<(15D+9c+8c_A)/16-11\varphi(h^2-3h+3)/12(h-2)\le(c_A-c_B)/2$ by simplifying and comparing and we get $0\le w_1^{a*}-w_1^{b*}\le(c_A-c_B)/2$. It is also easy to show that $\partial(w_1^{a*}-w_1^{b*})/(\partial h)\ge 0$ by taking the partial derivation of h from the above equation.

Corollary 4 Proof:

From Eq. (46) and Eq. (58) there are: $E(\pi_M^b)-E(\pi_M^a)=\dfrac{11h^2}{48}[(q_1^{b*})^2-(q_1^{a*})^2]$. Thus it is necessary to categorize q_1^{b*}, q_1^{a*} and discuss the same such that $G_A=3(2-h)(5D+3c-8c_A)/44(h^2-3h+3)$, $G_B=3(2-h)(5D+3c-8c_B)/44(h^2-3h+3)$, then we have $q_1^{b*}=\min[G_B,\varphi_B]$, $q_1^{a*}=\min[G_A,\varphi_A]$.

(1) When $\varphi_B\ge q_1^{a*}$, since $c_A>c_B$, it is easy to see that $G_B>G_A\ge q_1^{a*}$, so we have $q_1^{b*}\ge q_1^{a*}$, i.e. $E(\pi_M^b)\ge E(\pi_M^a)$.

(2) When $\varphi_B<q_1^{a*}$, since $q_1^{a*}\le G_A<G_B$, it is easy to see that $\varphi_B=q_1^{b*}$, so we have $q_1^{a*}<q_1^{b*}$, i.e. $E(\pi_M^b)<E(\pi_M^a)$.

Corollary 5 Proof:

$\partial k_1/\partial h\le 0$. $\partial k_1/\partial c\ge 0$ are easily obtained by partial derivation from
$k_1=q_1^{a*}=\min[3(2-h)(5D+3c-8c_A)/44(h^2-3h+3),\varphi_A]$, finish proof.

Corollary 6 Proof:

Obtained by subtracting Eq. (47) and Eq. (59):

$$E(\pi_S^b)-E(\pi_S^a)=\frac{h(q_1^{b*})\left(D+7c-8c_B\right)}{32}-\frac{3h^2(q_1^{b*})^2}{16}-\frac{h(q_1^{a*})\left(D+7c-8c_A\right)}{32}+\frac{3h^2(q_1^{a*})^2}{16}$$
$$\geq\frac{h(q_1^{b*})\left(D+7c-8c_B\right)}{32}-\frac{3h^2(q_1^{b*})^2}{16}-\frac{h(q_1^{a*})\left(D+7c-8c_B\right)}{32}+\frac{3h^2(q_1^{a*})^2}{16}$$
$$=\frac{(q_1^{b*}-q_1^{a*})}{32}h[D+7c-8c_B-6h(q_1^{b*}+q_1^{a*})]$$

According to Eq. (41) and Eq. (53), it is easy to obtain that $D+7c-8c_B-6h(q_1^{b*}+q_1^{a*})>0$. Therefore, it is also necessary to categorize and discuss the sizes of q_1^{b*} and q_1^{a*}. Similarly let: $G_A=3(2-h)(5D+3c-8c_A)/44(h^2-3h+3)$, $G_B=3(2-h)(5D+3c-8c_B)/44(h^2-3h+3)$, then we have $q_1^{b*}=\min[G_B,\varphi_B]$, $q_1^{a*}=\min[G_A,\varphi_A]$.

(1) When $\varphi_B\geq G_A$, there exists $\min[G_B,\varphi_B]\geq G_A\geq\min[G_A,\varphi_A]$ such that $q_1^{b*}\geq q_1^{a*}$, i.e., $E(\pi_S^b)>E(\pi_S^a)$.

(2) When $\varphi_B<G_A$, there exists $\varphi_B<G_B$, i.e., $q_1^{b*}=\varphi_B$, which gives:

$$E(\pi_S^b)-E(\pi_S^a)=\frac{h\varphi_B\left(D+7c-8c_B\right)}{32}-\frac{3h^2\varphi_B{}^2}{16}-\frac{h(q_1^{a*})\left(D+7c-8c_A\right)}{32}+\frac{3h^2(q_1^{a*})^2}{16}$$
$$=\frac{h}{32}[\varphi_B-\frac{D+7c-8c_B+\sqrt{(12hq_1^{a*}+8c_B-7c-D)^2+192q_1^{a*}h(c_A-c_B)}}{12h}]$$

Let $k_2=\dfrac{D+7c-8c_B+[(12hq_1^{a*}+8c_B-7c-D)^2+192q_1^{a*}h(c_A-c_B)]^{1/2}}{12h}$

If $k_2\leq\varphi_B<G_A$, there is $E(\pi_S^b)\geq E(\pi_S^a)$. If $k_2>\varphi_B$, there is $E(\pi_S^a)>E(\pi_S^b)$. Thus, synthesizing the above, when $\varphi_B<k_2$, there is $E(\pi_S^a)>E(\pi_S^b)$, and when $\varphi_B\geq k_2$, there is $E(\pi_S^b)\geq E(\pi_S^a)$.

Corollary 7 Proof:

From k_1-k_2 we have:

$$k_1-k_2=\frac{[(12hq_1^{a*}+8c_B-7c-D)^2+192q_1^{a*}h(c_A-c_B)]^{1/2}-(D+7c-8c_B-12hq_1^{a*})}{12h}$$
$$=\frac{16q_1^{a*}(c_A-c_B)}{[(12hq_1^{a*}+8c_B-7c-D)^2+192q_1^{a*}h(c_A-c_B)]^{1/2}+(D+7c-8c_B-12hq_1^{a*})}$$

and $\forall i\in\left(A,B\right)$, get:

$$D+7c-8c_B-12hq_1^{a*}\geq D+7c-8c_i-12h[3(2-h)(5D+3c-8c_A)/44(h^2-3h+3)]>0$$

i.e. $D+7c-8c_B-12hq_1^{a*}>0$, we can get $k_1>k_2$.

The partial derivatives with respect to h, c can be obtained by solving Equations k_1-k_2:

$$\frac{\partial(k_1-k_2)}{\partial h}=\frac{\dfrac{192(c_A-c_B)(q_1^{a*})^2\left[[(12hq_1^{a*}+8c_B-7c-D)^2+192q_1^{a*}h(c_A-c_B)]^{1/2}+D+7c-8c_B-12hq_1^{a*}\right]}{[(12hq_1^{a*}+8c_B-7c-D)^2+192q_1^{a*}h(c_A-c_B)]^{1/2}}}{\left[[(12hq_1^{a*}+8c_B-7c-D)^2+192q_1^{a*}h(c_A-c_B)]^{1/2}+D+7c-8c_B-12hq_1^{a*}\right]^2}$$

$$\frac{\partial(k_1-k_2)}{\partial c}=\frac{\dfrac{-176q_1^{a*}(c_A-c_B)\left[[(12hq_1^{a*}+8c_B-7c-D)^2+192q_1^{a*}h(c_A-c_B)]^{1/2}+D+7c-8c_B-12hq_1^{a*}\right]}{[(12hq_1^{a*}+8c_B-7c-D)^2+192q_1^{a*}h(c_A-c_B)]^{1/2}}}{\left[[(12hq_1^{a*}+8c_B-7c-D)^2+192q_1^{a*}h(c_A-c_B)]^{1/2}+D+7c-8c_B-12hq_1^{a*}\right]^2}$$

And because of $D+7c-8c_B-12hq_1^{a*}>0$, that means we can get $\dfrac{\partial(k_1-k_2)}{\partial h}>0$, $\dfrac{\partial(k_1-k_2)}{\partial c}<0$, and the proof is complete.