

A review of mathematical methods in energy management optimization

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ABSTRACT

Annual increases in power use need improved energy management. Several universities and research organizations have focused on pursuing energy efficiency and renewable energy to satisfy this stipulation. This study provides a thorough and organized systematic review of over 2000 operational research studies conducted between 2019 and 2023. In summary, this study explores potential innovations to enhance existing literature utilizing mathematical tools in energy management. Our systematic literature review indicates that geometric planning is a novel mathematical technique in energy management. Based on the specific problems, this paper discusses geometric planning and proposes its integration with other mathematical techniques.

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1. Introduction

Global warming has emerged as a prominent sustainability issue in contemporary civilization because of the significant contribution of greenhouse gas (GHG) emissions.^{1,2} In light of the current upsurge in energy use, the significance of energy management concerns in the context of sustainable development has been amplified. Energy management challenges include two fundamental components: power production and consumption. Academic researchers have devised a range of mathematical methodologies to address the optimization challenges linked to energy management targets effectively. These goals include improving energy conservation, controlling emissions, reducing costs, promoting energy efficiency, assessing demand patterns, maximizing the welfare of society, and attaining optimal power distribution. This intricacy is made even greater by the use of energy concepts. This paper primarily focuses on two energy management streams: energy generation and consumption. The first stream is energy efficiency, which specifically addresses operational aspects of power consumption. The second stream is renewable energy, representing a practical power generation approach.

The increase in environmental consciousness about more efficient items has led to a substantial growth in the energy efficiency of energy-intensive products.³ According to a study by Zhang et al.,⁴ a positive correlation exists between a product's energy efficiency level and the manufactured cost per unit. Energy efficiency is crucial in enabling manufacturers to gain a competitive advantage and access the global market. Additionally, it catalyzes the development of novel business models, advanced technology, and innovative services.⁵⁻⁷ The manufacturer is seeking to establish the product's selling price and energy efficiency level. When making decisions about efficiency measures, it is important for manufacturers to carefully consider the costs and benefits, taking into account the potential increase in demand that may result.⁸

Researchers have found that governments are exploring incentive energy programs as a means to promote social welfare and foster energy conservation.^{9,10} Governments implement these programs through a variety of methods, including providing tax deductions or subsidies to manufacturers and customers for energy-efficient goods.⁷ In 2007, Shandong, Henan, and Sichuan provinces implemented an incentive program for rural home appliances, serving as evidence of their commitment to promoting energy efficiency. These products need to comply with both safety regulations and national

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energy efficiency standards at the first or second level. In 2012, the Chinese State Council introduced a \$4.1 billion incentive program to foster awareness and adoption of energy conservation practices. The marketing initiative was directed at a range of domestic appliances, including air conditioning units, refrigeration units, washing machines, flat-screen televisions, and water heating systems.¹¹

As per Bilgen et al.¹² the usage of fuel in large, centralized power plants has a notable effect on the quantity of carbon dioxide emitted into the atmosphere in various countries. The objective of governments is to encourage the expansion of the energy sector in a manner that both satisfies the energy demand and reduces pollution, emissions, and consumption to the greatest extent possible. Based on the research conducted by Zhang et al.¹³ it has been determined that renewable energy can make a substantial contribution towards the reduction of carbon emissions in nations. Moreover, it holds exceptional significance in addressing the social and economic aspects of sustainable development. The construction of sustainable energy infrastructure in Germany has resulted in significant progress being achieved in the prevention and mitigation of climate change. There is a significant example that refers to the development of microgrids that are powered by renewable energy sources in different locations.¹⁴ The incorporation of microgrids into the electricity grid does not always result in a decrease in the amount of greenhouse gas emissions and carbon intensity, according to research carried out in 2020 by Papageorgiou and his colleagues.¹⁵ In light of the issues above, this article provides an overview of the mathematical methodologies used in the existing body of research about energy management. Geometric programming (GP) emerges as a prominent research topic within this domain, to address the concerns above. These issues will be further on in the following sections. The present paper aims to address the following research questions:

- (1) What fresh study has been conducted in the domain of energy management focusing on the two aspects of energy generation and consumption?
- 2) According to a systematic study in the area of energy management, what is the correlation between keywords in prior research?
- 3) Considering the analysis of prior research and the recognized requirements of the energy management sector, which mathematical tool is lacking and how may they be addressed?

Furthermore, this review article's innovations and unique characteristics compared to other publications will be followed in answering the research questions.

The main advantage of this work is identifying practical but unknown mathematical tools in the energy management literature. Introducing these tools provides a context for solving more complex problems in the field of energy production and consumption with the unknown mathematical tool, geometric planning. Furthermore, this article offers a concise preface to geometric planning and a summary of the publications that employ it, so presenting a more effective approach to its application in forthcoming energy management articles.

This study also provides an overview of the theoretical underpinnings and specific concepts of geometric programming, to inform future research endeavors in the field of energy management. The review methodology involves using keywords related to energy management and mathematical tools. The following elements are among the primary contributions of this research: (1) An examination of renewable energy, which is one of the most critical concepts in the domain of energy generation management, (2) A review of energy efficiency, one of the chief concepts in energy consumption Management (3) The bibliometric analysis of certain mathematical instruments in energy management using the VOSviewer software. (4) The introduction of geometric programming fundamentals as a research gap of utilized mathematical tools in the energy management field. (5) The extraction of the key research gaps and establishing guidelines for future research directions.

As previously indicated the escalating rise in energy consumption has made energy management an essential need in contemporary society. This need may be analyzed in two aspects: consumption and production. In the realm of production, the utilization of renewable energy sources and, in the domain of consumption, the use of high energy efficiency appliances serve as the fundamental principles of energy management. This article provides a detailed review of the research literature in these two specific areas. A systematic review and bibliographic presentation using VOSviewer software are used to identify more fundamental principles in the energy management area and the required mathematical tools for their clarification.

Given the intricate nature of energy management resulting from the multitude of influential elements, it is essential to represent these concerns of energy generation and consumption using non-linear programming. Conversely, the multiplicity of decision-makers in this domain and the interdependence of their decisions need the use of the game theory methodology for problem-solving. Indeed, the inherent unpredictability of natural resources like sun and wind, among others, as sources of renewable energy, necessitates the use of robust optimization and stochastic optimization techniques. The findings of this study, within the context of prior literature review, indicate that the game theory approach has been extensively utilized in the field of energy management. It is considered one of the most commonly employed methods in energy management papers. However, geometric planning has been overlooked. Given the complex and non-linear nature of energy-related problems, it is imperative to incorporate geometric planning into the work. This integration should be done in conjunction with other mathematical methods and tailored to the specific conditions of the problem, whether it is interactive, dynamic, or probabilistic.

The subsequent portions of the research are outlined as follows. The second section provides a concise overview of energy management ideas, which comprise the two fundamental components of power generation and consumption. Section 3 offers an exhaustive review of the mathematical methodologies used in energy management research. Section 4 proposes the inclusion of geometric programming as a research gap in the existing literature, to guide future efforts. The present paper has reached its conclusion, and the findings are briefly outlined in Section 5.

2. A brief overview of energy management key concepts

This section primarily focuses on two subfields within energy management, namely power generation and consumption. It aims to emphasize the primary areas of potential research: (i) renewable energy, which represents a practical approach in the realm of power generation, and (ii) energy efficiency, which concerns the operational aspects of power consumption.

2.1. Renewable energy

The upsurge in global energy demand and the mounting political pressure to reduce carbon emissions have led to a surge in the adoption of renewable energy sources.¹⁶ As a result of the pressure that rising carbon emissions are placing on conventional energy sources, there is a growing trend toward renewable energy sources such as photovoltaic (PV) power, wind power, and hydropower. Global carbon emissions have pressured traditional power sources.¹⁷ The exploration and implementation of power generation systems utilizing renewable energy sources have garnered significant interest among professionals and researchers alike. Given its potential as a feasible alternative to traditional energy sources, this area of research has garnered noteworthy interest and scrutiny in recent times. The study conducted by Wang et al.¹⁸ utilizes a non-cooperative game theoretical framework to examine the impact of demand response on the adoption of distributed photovoltaics (PV) among smart home users in the energy market.

Tushar et al.¹⁹ investigate a mixed-strategy game that enables participants to mitigate anticipated electricity demand by flattening future demand curves. The authors conduct a comparative analysis between their approach and non-cooperative centralized and decentralized electric vehicle (EV) charging management games. Van Ackooij et al.²⁰ propose a comprehensive model that specifically addresses the interaction between microgrids and conducts numerical analysis for the bilevel energy management optimization problem and a single leader problem with multiple followers. To facilitate the adoption of solar energy, Fang et al.²¹ propose a collaborative approach between investment firms and homeowners, wherein companies offer to install rooftop solar panels in return for monetary compensation from electric vehicle owners. The assessment of stakeholder participation preferences is conducted through evolutionary game strategies. A study conducted by Akbari-Dibavar et al.²² examines the application of a hybrid robust-stochastic optimization model in smart home energy management. The objective of the study is to mitigate market price uncertainty associated with worst-case photovoltaics in both day-ahead and real-time markets.

Dai et al.²³ propose a dynamic pricing strategy that utilizes the Stackelberg game to determine the pricing of electric vehicle charging stations equipped with photovoltaic systems. This methodology considers the inherent uncertainty associated with power fluctuations before and after the operation. Lin et al.²⁴ employ the Newsvendor and Stackelberg methodologies to investigate the deployment of distributed photovoltaic power generation. This approach aims to mitigate manufacturing expenses and promote the adoption of renewable energy sources. Specifically, the study explores the utilization of solar energy in product production and the subsequent sale of surplus electricity to generate income.

The study conducted by Erol and Filik²⁵ examines the management of energy sharing in a microgrid characterized by internal pricing determined by entity utilities. The researchers also explore the dynamic nature of prosumer roles, which can change as buyers or sellers engage in a single-leader multi-follower Stackelberg game. Luo et al.²⁶ propose a decentralized trading strategy utilizing game theory to examine the impact of distributed ownership of energy resources on participants in the peer-to-peer energy market sectors. M. Zhang et al.¹⁷ present an optimal energy and reserve scheduling approach for a renewable-dominant power system, considering the complexities associated with uncertainty and robust optimization. They present a scheduling problem about energy and reserves within a power system that predominantly relies on renewable forms of energy. The objective of their problem is to attain a balanced state between risk aversion and optimality, while simultaneously ensuring flexibility and stability.

2.2. Energy efficiency

The recent surge in energy use has further emphasized the importance of energy efficiency issues in the production of energy-intensive products within the context of sustainable development. Therefore, there has been a significant rise in the energy efficiency of these items, mostly attributed to the increasing environmental awareness associated with energy-efficient products. An enhancement in energy efficiency is followed by a rise in the company's total cost per unit of production. According to Safarzadeh et al.⁶, the promotion of energy efficiency fosters the emergence of novel business models, advanced technology, and services, therefore providing firms with a competitive advantage and facilitating entry into previously untapped areas. According to Safarzadeh and Rasti-Barzoki's⁵ research from 2020, a substantial amount of research has been carried out on the subject of energy efficiency. In the context of a monopoly, W. Zhou and Huang²⁷

examine two distinct contract forms for energy-saving products, employing the limited funds provided by the government. Through the use of game theory, Zhou et al.²⁸ examine how energy performance contracting affects two firms engaged in a Cournot competition.

Safarzadeh and Rasti-Barzoki²⁹ analyze the competition between efficient and inefficient manufacturers under government oversight through tax and subsidy policies. Drawing upon the Bertrand model, Safarzadeh and Rasti-Barzoki³⁰ propose the implementation of a multi-stage sustainable supply chain encompassing the government, a manufacturer, and an energy supplier. An analysis conducted by Huang et al.¹¹ examines the optimal government subsidies for energy-efficient products in a market with two competing firms. Based on the time and energy requirements, uit het Broek et al.³¹ examine energy-conservation methods for a manufacturing system structured as an M / G / 1 queue. The sustainable supply chain proposed by Safarzadeh and Rasti-Barzoki⁵ is constructed using the Cournot competition model, incorporating both an energy supplier and an energy-efficient producer. Rasti-Barzoki and Moon³² use the Stackelberg game theoretical framework to analyze the process by which automakers determine the cost and energy efficiency of their vehicles within a duopolistic supply chain consisting of one energy source and two producers. Through the use of a multi-stage game, Safarzadeh, Rasti-Barzoki, Hejazi, and Piran⁷ analyze energy efficiency.

Yan and colleagues³³ developed a new linearised energy-conserving Crank-Nicolson finite volume element method to solve the enhanced Boussinesq equation. W. Wang et al.³⁴ examine the output feedback control of an asymmetric hydraulic servo system for energy reserves. They propose analytical modeling of a manufacturer to enhance the optimized payout. In the context of fluctuating energy prices, Chargui et al.³⁵ investigate berth allocation, quay crane assignment, and scheduling. To resolve this issue, they develop an exact decomposition procedure. Bai et al.³⁶ suggest a two-part tariff contract to facilitate energy-saving investments and supply chain coordination. The paper examines the welfare of society, consumer benefits, and profits of the supply chain. Fatemi et al.³⁷ employing a combination of geometric programming and game theory, utilizing the Stackelberg game framework, devised an analytical method for sensitivity analysis. This method examines the influence of the tax on manufacturer decisions, considering energy efficiency and price-dependent demand. Additionally, they analyze the impact of energy efficiency on production costs.

3. Systematic review

Optimizing loads, consumption, energy storage, user settings, power resource conservation, integration, planning, and management of many operations necessitates using diverse mathematical methodologies due to their inherent complexity. The main objective of this work is to analyze the mathematical models published in the literature on energy management and pinpoint areas where more research is needed. This section utilizes the Visualization of Similarities (VOSviewer software package) to generate a graphical depiction of the bibliographic content published in OR/MS journals of EVISE. The purpose is to examine the relevant literature, identify any gaps, and assess potential contributions to the existing literature. VOSviewer is a software application designed to create and display bibliometric networks. It automatically distributes the nodes in a network into clusters. In computing, a cluster is a collection of closely associated nodes. Every individual node within a network is allocated strictly to a single cluster. A resolution parameter directly determines the number of clusters.

Fig. 1 designates each term to a separate cluster. The clusters are formed by the number of co-occurrences between words in the relevant literature, and color variations indicate distinct research clusters. Figure 1 depicts the key concepts important to the subject and those that are secondary. Figure (1) demonstrates the key concepts pertinent to the subject and those that are peripheral. This Figure examines and compares several mathematical methodologies and concepts that are present in the current literature on energy management, including game theory, as well as the connections between them.

Fig. 2 illustrates the density with a rainbow density visualization in VOSviewer. Variations in color range from light blue to green and yellow to red. The closer the color of a point approaches red, the higher the number of items in its vicinity and the larger the weights assigned to those items. The closer the color of a point approaches light blue, the smaller the number of items in its vicinity and the lower the weights of those items. **Fig. 3** displays labels and circles representing the key concepts, while the lines denote the connection between the two items. In network visualization, the relative size of a node to other nodes reflects its importance, with bigger nodes being more notable. The evaluation is not a classification of the content level of the items, but rather a bibliometric measure of the importance of the items within their respective cluster. The colors of the allotted items in **Fig. 4** illustrate cluster density visualization and help to distinguish the clusters. Like the visualization of item density in **Fig. 5**, the visualization of cluster density in **Fig. 4** additionally considers the weight of the items into account. The weight that is ascribed to a specific cluster's color is determined by the number of items related to that cluster in the vicinity of the point. As the weight of the item increases, its label and circle become larger.

Fig. 5 illustrates the representation of item density, where labels match the ideas and the color of an area on the map indicates the item density at that point. The item density display in **Fig. 5** does not depict the correlation between words; rather, it demonstrates the words that are well integrated and those that are frequently studied separately. By using a time-based overlay presentation that charts pertinent terms from 2019 to 2022, **Fig. 6** emphasizes the emerging research topics.

It is crucial to analyze the primary terms in published papers and establish their correlation with the year of publication to differentiate innovative research themes from established ones. In Fig. 6, the color of a node represents the time at which a subject is added to the network. Geometric programming is conspicuously lacking in the relevant literature on this subject.

The bibliographic connections between the items are approximately represented by the distance between them in Fig. 7. In general, the stronger the relationship between the two items, the closer they are. Objects are arranged in these maps using a distance-based method by VOSviewer, and the distance between objects is more important than the cluster to which they belong when assessing the structure of the scientific field in this visualization. In this visualization, geometric programming and game theory are shown as distant entities. Significantly, it is important to include geometric programming (GP) into the game theory methodology to account for the non-linear nature of the production and demand cost functions, as well as the interaction among players in the energy chain, hence enhancing the realism of modeling. The web-based program Wordclouds (<https://www.wordclouds.com/>) is used to create the word cloud shown in Fig. 8. This word cloud is a graphical representation of text data that shows the most often occurring terms in this particular context. A widely used information visualization technique, word clouds enable the examination of keywords in this subject and the determination of the academic focus and attention of the writers.

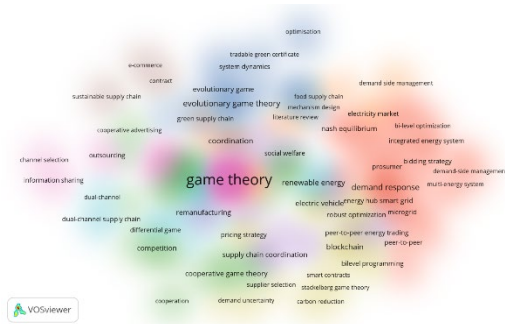


Fig. 1. Topic mapping analysis.

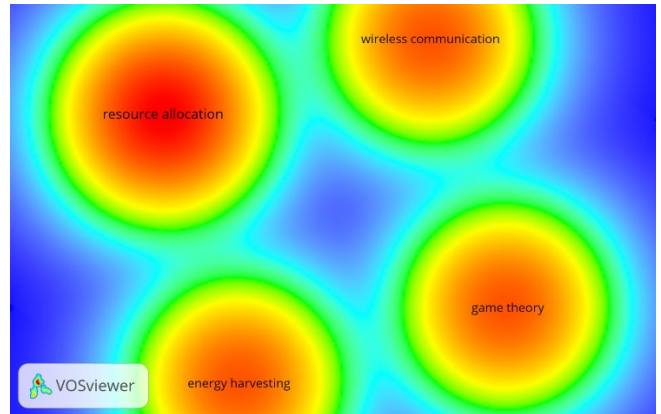


Fig. 2. Rainbow density visualization.

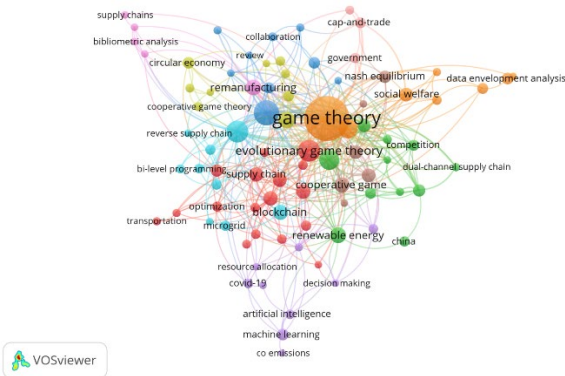


Fig. 3. Keywords co-occurrence network.

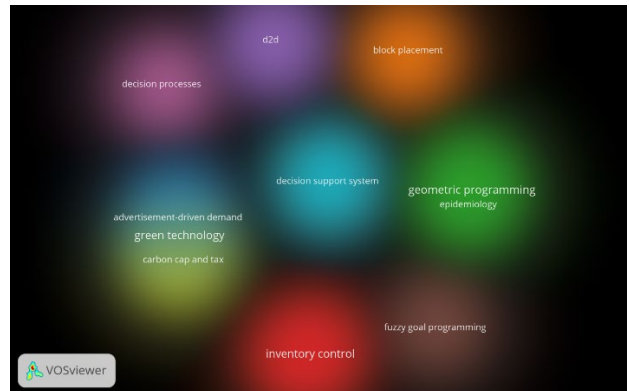


Fig. 4. Cluster density visualization.

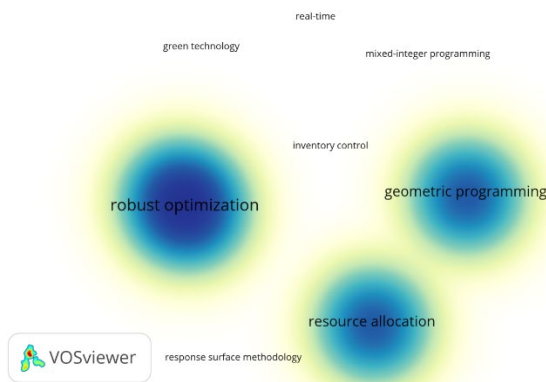


Fig. 5. Item density visualization.

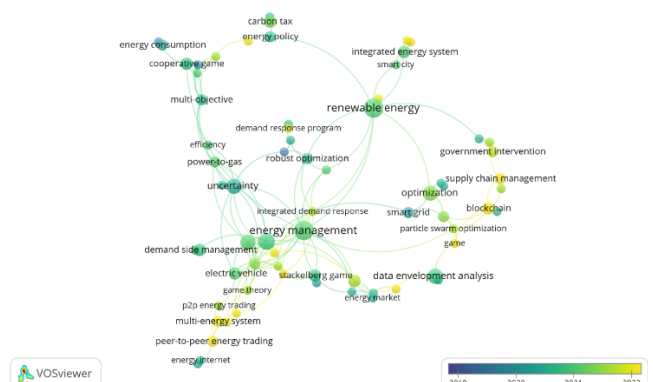


Fig. 6. Keyword analysis during 2019–2022.

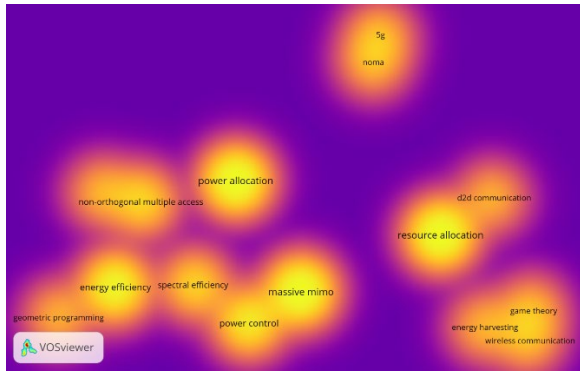


Fig. 7. Bibliographic map.



Fig. 8. The word cloud.

The present study analyzes the existing research gaps in the prior literature. Given the complex and non-linear characteristics of the challenges in practical energy management, this research gap is unique and not found in the existing literature. This study has discovered a second research gap: integrating geometric programming (GP) into game theory to investigate the interactions between decision-makers and the non-linear characteristics of production and demand cost functions. This integration will improve the authenticity of the modeling.

4. Geometric programming: A gap in energy literature

Drawing from prior sections and systematic reviews of the energy management literature, this paper identifies geometric programming as a significant deficiency in the current literature on energy management. This section thoroughly introduces this potent mathematical approach for nonlinear problems, building upon prior research. Energy management models with exponential demand elasticities need the use of geometric programming (GP), a powerful mathematical approach for addressing non-linear dynamics. Geometric programming (GP) is a very efficient mathematical optimization method used to solve many real-world issues that include non-linear exponential functions³⁸. In contrast to Duffin and Peterson³⁹, who focus on a single decision-maker and do not integrate game theory to represent participant interaction, this study offers important additions to the GP literature. The approach of geometric planning has been used by others to effectively model and solve a range of intricate issues inside novel frameworks. Previous studies, such as Shen et al.⁴⁰, have further developed this mathematical technique. A helpful and adaptable solution to solving the sum of generalized polynomial ratios issue under generalized polynomial restrictions is provided by Shen et al.⁴⁰. In this paper, we provide an overview of basic transformation and contraction methods, as well as the solutions to many geometric programming problems.

In 2005, Sadjadi et al.⁴¹ used Generalized Programming (GP) to determine the optimal solution for a comprehensive model that integrates inventory, marketing, and production factors. This mathematical model determines the optimal dimensions of the manufacturing lot, marketing budget, and selling price of the product. In their study, Sadjadi et al.⁴² use Fuzzy geometric programming to demonstrate a new possibilistic model that effectively integrates expert viewpoints into pricing and marketing planning models. 2015 research by Sadjadi et al.⁴³ examines the use of geometric programming methods to improve lot-sizing, pricing, and marketing tactics, considering production needs such as process flexibility and dependability, to enhance profitability for firms. Using a robust optimization framework, Lim et al.⁴⁴ propose identifying the optimum bundle of price and order quantity for a store in a two-stage supply chain. The proposed model incorporates parameter uncertainty in both demand and purchase cost functions. More precisely, the model represents demand as a declining power function of the product's price, while the cost of purchasing each unit remains a falling power function of both the amount ordered and the demand. Ruby and colleagues⁴⁵ use geometric programming to assign subchannels and power supplies of the base station to users. To achieve this objective, the issue may be divided into two subproblems: subchannel allocation and power-loading. Subchannel allocation manages the distribution of power to the data stream of each user on their respective subchannels. Using an extended geometric programming method, Yaghin⁴⁶ created a complete model that combines multi-site production and marketing to manage revenue in aggregate production planning problems inside a garment supply chain. Multitask learning is employed by Hayhoe et al.⁴⁷ to obtain compartmental discrete-time epidemic model parameters from a variety of data sources. This model develops control methods for human mobility restriction that reduce the costs of non-pharmaceutical intervention and epidemic transmission. This paper employs geometric programming to resolve this nonlinear optimal control problem through rapid polynomial-time calculations. An optimization approach to optimize the lower bound in a multi-tier heterogeneous cellular network is presented by Z. Liu et al.⁴⁸ This result is achieved by stochastic geometric programming, which involves adjusting the transmitting power and density of the base station. The functions are non-linear and must be solved using geometric programming (GP) due to the exponential growth of manufacturing costs and demand in real-world problems. GP is a mathematical optimization technique that is effective in addressing a variety of real-world issues associated with non-linear exponential problems.⁴⁹⁻⁵¹

The geometric programming method is summarized in Fig. (9), which is used to identify the optimal solutions to the problem.

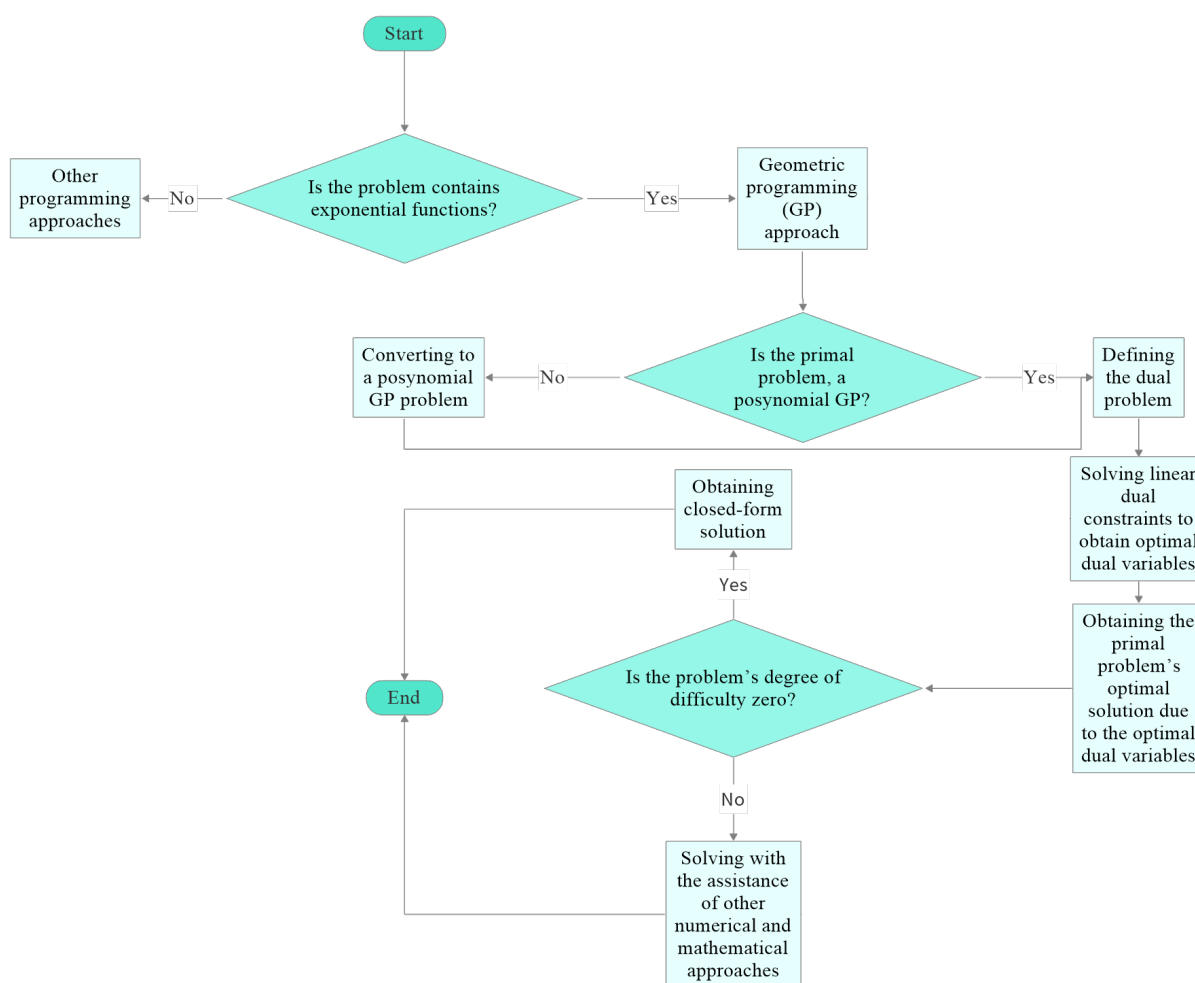


Fig. 9. GP flowchart to obtain optimal solutions.

5. Conclusion

5.1. Concluding remarks

Positive results of successful energy management include reduced energy bills, lower load on the electric energy system, and greater energy efficiency. This study investigates the mathematical methodologies used in studies and research aimed at improving energy management. In summary, geometric programming is identified as a gap in the existing literature, and this study offers a comprehensive examination of this method by drawing upon pertinent scholarly sources.

5.2. Key findings

This paper's main results show that non-linear programming is the way to go for representing energy production and consumption issues, which is crucial since energy management is complex due to the many contributing factors. On the other hand, game theory is a necessary tool for addressing problems in this sector due to the large number of decision-makers and the interconnected nature of their choices. Indeed, stochastic optimization and resilient optimization are required due to the intrinsic unpredictability of renewable energy sources such as the sun and wind. This study's results, when placed in the context of the reviewed literature, show that energy management has made heavy use of the game theory method. In studies on energy management, ranks high as one of the most popular methods. But we haven't paid enough attention to geometric planning. Geometric planning must be part of the process since energy-related issues are complicated and non-linear. It is important to combine this integration with other mathematical approaches and adapt it to the particular circumstances of the issue at hand, whether it be interactive, dynamic, or probabilistic.

5.3. Suggestions for future studies

To guide future research endeavors in energy management modeling, this article examines mathematical approaches used for the analytical optimization of energy management decisions. This paper addresses key unique approaches based on geometric programming as a practical mathematical method for handling non-linear energy management problems. There is a notable absence of geometric programming in the pertinent literature on this subject, which can be utilized in conjunction with game theory to address conflicts between energy management decision-makers in nonlinear problems. Additionally, it is suggested that a hybrid of optimal control and geometric programming be employed to examine the dynamic nature of energy management issues. Nevertheless, this combination exacerbates the complexity of the problem-solving process. Another recommendation is to employ stochastic programming coupled with geometric programming to address the challenges associated with energy management that are characterized by uncertain circumstances and an exponential nature.

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