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Fuzzy portfolio optimization using conditional drawdown at risk: Empirical evidence on selective companies in the Tehran Stock Exchange

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A B S T R A C T

This article introduces an innovative fuzzy-based approach for developing a comprehensive portfolio optimization model that effectively accounts for inherent uncertainty while incorporating the investor's unique perspective on the dynamic stock market. The multi-objective optimization framework employs Conditional Drawdown at Risk to enhance investor flexibility in determining risk tolerance and optimal investment strategies tailored to specific needs. The research is notable for its pioneering use of intelligent methods to systematically collect valuable data from the Tehran Stock Exchange under fuzzy uncertainty. It incorporates important constraints such as cardinality and ceiling and floor limits for each investment period, allowing for a detailed analysis of various stock market scenarios and potential future outcomes. A case study is conducted with 25 diverse assets from the top five industry groups based on profit per share, from which five shares are thoughtfully selected to effectively demonstrate the model's unique effectiveness. The analysis rigorously assesses the model's performance in real-world conditions, highlighting the importance of accurately understanding the current stock market outlook and trends. To validate the model, the research compares results with a portfolio constructed under similar conditions of certainty and risk. The findings indicate that portfolios created under certainty yield significantly higher values, suggesting that successful portfolio construction is heavily influenced by the prevailing market conditions experienced by investors.

1. Introduction

Portfolio theory and its associated subjects are considered some of the most extensively analyzed fields within economic and financial research. The widely adopted mean-variance framework introduced by Markowitz (1952, 1959) emphasizes the necessity of mitigating the risk associated with a chosen asset portfolio while achieving a specified return and optimizing available capital. This approach examines the trade-off between risk and return through the lens of mean and variance metrics (Ghanbari et al., 2023). According to Markowitz's model, investors have the option to either optimize expected portfolio returns for a defined level of risk or to minimize investment risk by decreasing the variance of the portfolio for a specified target return (Khosravi et al., 2024).

Markowitz's principal contributions are founded upon important assumptions concerning probability distributions and Von Neumann–Morgenstern utility functions. Nevertheless, as the complexity of portfolio optimization issues escalates, addressing quadratic programming involving a dense covariance matrix becomes increasingly difficult. (Eskorouchi et al., 2022). Researchers have suggested different approximation methods to reduce these computational challenges (Sharpe, 1967, 1971; Stone, 1973). The implementation of the index model contributes to the reduction of computational demands by integrating "factors" that influence stock prices (Perold, 1984; Sharpe, 1964).

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Notwithstanding these advancements, the Markowitz model is still considered the most comprehensive framework currently available. Although the limitations associated with employing a quadratic approximation for utility functions are recognized, the formulation of effective portfolio construction methods has largely been impeded by computational challenges. There is a need to improve both the computational and theoretical aspects of Markowitz's model, Konno & Yamazaki (1991) proposed a portfolio optimization framework that employs a mean-absolute deviation risk function as a substitute for Markowitz's standard deviation risk function. This model effectively addresses the challenges associated with the traditional Markowitz framework while retaining its benefits over equilibrium models. Notably, it enables formulation as a linear programming problem, thereby facilitating the resolution of large-scale portfolio optimization challenges (Zarezade et al., 2024).

Conventional portfolio optimization models often operate under the assumption that future market conditions can be reliably forecasted using historical data. However, this premise becomes problematic in highly volatile financial markets, regardless of the accuracy of historical data. To quantitatively address the uncertainties inherent in decision-making when faced with imprecise information, it is essential to reconsider this approach, Zadeh (1978) introduced the concept of fuzzy logic. Fuzzy set theory has found extensive applications in both linear and nonlinear optimization problems (Björk, 2009); Chen, 2004; Liu, 2005; Vijayan & Kumaran, 2009). In situations where parameters are uncertain or unclear, the expected returns of the portfolio will similarly display a level of ambiguity. As a result, the portfolio optimization context can be characterized as a fuzzy portfolio optimization problem. The primary objective of this study is to empirically evaluate the potential advantages of integrating fuzzy logic and Conditional Drawdown at Risk (CDaR) into the portfolio construction process. Fuzzy logic presents a robust framework for modeling the inherent imprecision and uncertainty present in financial markets, whereas CDaR provides a comprehensive risk measure that takes into account both the magnitude and duration of potential losses (Zarezade et al., 2024). By combining these two methodologies, we seek to create a more realistic and effective approach to portfolio selection. To fulfill our research objectives, we focus on a selection of companies listed on the Tehran Stock Exchange (TSE). Through meticulous identification and assessment of these companies based on various financial indicators, we develop a fuzzy portfolio utilizing a comprehensive range of membership functions and fuzzy inference rules. Subsequently, we evaluate the performance of this fuzzy portfolio against a specific portfolio optimization technique.

This study contributes several advancements to the existing literature. First, it expands the application of fuzzy logic and Conditional Drawdown at Risk (CDaR) in portfolio optimization, providing valuable insights into their effectiveness in the context of emerging markets. Second, it offers empirical evidence regarding the performance of fuzzy portfolios when compared to traditional methods, highlighting both the potential benefits and limitations of employing a fuzzy logic-based approach. Lastly, it presents practical implications for investors and portfolio managers aiming to enhance their investment decision-making processes through more adaptive and robust strategies. This research addresses current gaps by specifically investigating the performance of fuzzy portfolio optimization using CDaR within the TSE, incorporating the Jiménez method for fuzzy analysis to improve the optimization process. Furthermore, a targeted technique for stock selection from the available options is employed, thereby enhancing the relevance and applicability of the findings.

The structure of this paper is organized as follows: Section 2 provides a comprehensive review of the relevant literature, examining contemporary portfolio optimization methods and the rationale for utilizing fuzzy logic and CDaR. Section 3 details the methodology, encompassing data collection, fuzzy modeling, and portfolio construction processes. Section 4 presents the results of the empirical analysis, followed by a discussion of these findings in Section 5. Finally, Section 6 concludes the paper with a summary of key findings, implications, and recommendations for future research.

2. Review of Literature

The domain of portfolio optimization has experienced considerable evolution, particularly through the incorporation of fuzzy logic and sophisticated risk measures such as Conditional Value-at-Risk (CDaR). This literature review assembles essential contributions to the advancement and implementation of fuzzy portfolio optimization techniques, underscoring their importance in managing uncertainty and improving financial decision-making.

Initial contributions to fuzzy portfolio optimization can be traced to Ammar & Khalifa (2003), who developed a fuzzy model grounded in Markowitz's mean-variance framework. This model incorporated the notion of fuzziness into portfolio selection, facilitating a more sophisticated method for addressing uncertainty in asset returns and risks. Building on this foundation, Fei (2007) Explored the optimal strategies for consumption and portfolio selection through a Merton-style framework, differentiating between ambiguity and risk an essential viewpoint that highlights the intricacies of investor decision-making under conditions of uncertainty.

Further advancements were made by Ammar (2008), who reconceptualized the portfolio optimization problem as a multi-objective quadratic programming challenge, utilizing fuzzy numbers for both objectives and constraints. This methodology provided enhanced flexibility and a more accurate representation of investor preferences. (Gupta et al. (2008) also advanced the discussion by employing fuzzy set theory to formulate a semi-absolute deviation model, which emphasizes the practical requirements of investors dealing with uncertainty in asset returns.

Vercher (2008) broadened the discourse by applying a semi-infinite programming technique to tackle portfolio selection problems characterized by fuzzy returns. This methodological advancement underscored the efficacy of semi-infinite programming in navigating the intricacies associated with fuzzy portfolio decision-making. Concurrently, Chen & Huang (2009) adopted cluster analysis alongside the Sakawa et al. (1993) method, which is aimed at optimizing equity mutual funds while accommodating fuzzy return rates and risks. Their work exemplifies the versatility of fuzzy logic in diverse financial contexts.

The integration of fuzzy logic with CDaR represents a significant advancement in portfolio optimization methodologies. CDaR enhances classical risk measures, such as Value at Risk (VaR), by incorporating the severity and duration of potential losses into the assessment. Kwon and Law (2014) were pivotal in introducing CDaR as a risk metric in portfolio selection, emphasizing its effectiveness in encapsulating downside risk and refining risk management strategies. By enabling a more holistic evaluation of risk, CDaR allows investors to consider both the intensity and length of drawdowns, which is crucial for effective portfolio construction.

Li et al. (2020) conducted a comparative analysis of fuzzy portfolio optimization versus traditional approaches within the Chinese stock market, revealing its superior performance in managing market uncertainties and delivering enhanced risk-adjusted returns Huang et al. (2021) further demonstrated the efficacy of combining fuzzy logic with CDaR in portfolio optimization. Their proposed model not only utilized CDaR as the risk measure but also showcased substantial advantages over traditional mean-variance portfolios. This integration promotes a more adaptive and realistic framework for portfolio selection, taking into account the imprecision inherent in financial data and agent preferences.

Empirical studies have validated the effectiveness of fuzzy portfolio optimization across various market environments. Similarly, Wu et al. (2021) explored the application of fuzzy portfolio optimization in the Taiwan stock market, illustrating its capacity to improve portfolio performance amid volatile conditions.

In light of the existing literature, this research seeks to bridge the gap by examining the performance of fuzzy portfolio optimization utilizing CDaR specifically within the TSE. Among its innovations, this study employs the Jiménez method for fuzzy analysis, which further refines the optimization process. Additionally, a specialized technique for selecting stocks from the available universe is implemented, enhancing the relevance and applicability of the findings.

In summary, the literature demonstrates that fuzzy portfolio optimization and CDaR represent powerful approaches for addressing the inherent uncertainty and complexity in financial markets. By incorporating fuzzy logic, these methodologies enhance traditional portfolio management strategies, offering investors robust tools to navigate the challenges of volatility and imprecision. Through this research, we contribute to the understanding and application of these innovative techniques in emerging markets.

3. Preliminaries

This section contains two subsections that clarify the mathematical models and data utilized in the study. The explanations offered are thorough, to improve the reader's comprehension and encourage their involvement with the topic.

3.1 Mathematical Models

This section commences with an introduction to the principles related to evaluating the potential capital loss risk within conditional risk as part of the portfolio optimization process. Following this, a model is introduced for the optimization of the stock portfolio, particularly focusing on parameters characterized by uncertainty. To tackle this, the fuzzy programming model based on Jimenez's approach is utilized, culminating in the presentation of the ultimate model. The calculation of the rate of return for each stock is conducted using equation (1).

$$r_t = \text{Ln}\left(\frac{p_t}{p_{t-1}}\right) \tag{1}$$

In the aforementioned equation, p_t and present the prices of stocks during periods t and $t-1$, correspondingly.

Definition 1. Conditional Drawdown at Risk (CDaR)

The CDaR metric was initially introduced by Chekhlov et al. (2004) and is regarded as a relatively new risk measure with unfavorable characteristics. Its behavior closely resembles that of the value-at-risk conditional risk measure, which was initially explored by Rockafellar and Uryasev, (2000). Krokhmal et al. (2005) performed a comparative analysis of these two risk measures, in addition to the capital-at-risk conditional risk and value-at-risk conditional risk measures, within the context of risk hedging funds. Their study findings suggested that the CDaR measure is more cautious, whereas the value-at-risk conditional risk measure provides greater flexibility.

The CDaR model can be expressed as follows; Let $w(x,t)$ be the uncompounded portfolio value at time t and suppose that $x = (x_1, x_2, \dots, x_n)$ is the weights of assets in the portfolio, thus the drawdown function at time t is defined by:

$$f(x,t) = \max_{0\le\tau\le t}\{w(x,\tau)\} - w(x,t) \tag{2}$$

Suppose that r_{it} is the rate of return of i-th asset in j-th trading period. The uncompounded portfolio value at time j equals:

$$w(x,j) = \sum_{i=1}^{n}\left(1 + \sum_{t=1}^{j} r_{it}\right)x_i \tag{3}$$

Then, the drawdown function at time j can be expressed as below.

$$f(x,j) = \max_{1\le k\le j}\left\{\sum_{i=1}^{n}\left(\sum_{t=1}^{k} r_{it}\right)x_i\right\} - \sum_{i=1}^{n}\left(\sum_{t=1}^{j} r_{it}\right)x_i \tag{4}$$

Considering that CDaR is the average of the worst-case drawdowns observed in the considered sample path, we can define CDaR as follows:

$$CDaR_\alpha(x,\eta) = \eta_\alpha + (1-\alpha)^{-1}\sum_{j=1}^{J}[f(x,j) - \eta_\alpha]^{-} \tag{5}$$

where η represents the threshold drawdown level which is exceeded by $(1-\alpha)J$ drawdowns, and $\alpha \in [0,1]$ denotes the confidence level. The CDaR model can also be represented as:

$$CDaR_\alpha(x,\eta) = \eta_\alpha + \frac{1}{(1-\alpha)J}\sum_{j=1}^{J} max\left\{0, \max_{1\le k\le j}\left[\sum_{i=1}^{n}\left(\sum_{t=1}^{k} r_{it}\right)x_i\right] - \sum_{i=1}^{n}\left(\sum_{t=1}^{j} r_{it}\right)x_i - \eta_\alpha\right\} \tag{6}$$

Eqs. (7-12) demonstrate the existence of a linear correlation between the portfolio optimization model and the value-at-risk measure

The linear relationship CDaR is as follows:

$$\min \eta + \frac{1}{(1-\alpha)}\frac{1}{J}\sum_{j=1}^{J}(y_j) \tag{7}$$

Subjected to

$$\sum_{i=1}^{n} \mu_i x_i = \mu_p \tag{8}$$

$$y_j \ge \left\{\sum_{i=1}^{n}\left(\sum_{t=1}^{k} r_{it}\right)x_i\right\} - \left\{\sum_{i=1}^{n}\left(\sum_{t=1}^{j} r_{it}\right)x_i\right\} - \eta \tag{9}$$

$$y_j \ge 0 \tag{10}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{11}$$

$$x_j \ge 0. \quad i = 1.2.\dots n \tag{12}$$

In this mathematical framework, Eq. (7) represents the objective function, which measures the portfolio's value at risk. Eq. (8) describes the relationship that equates the portfolio's return with the investor's expected return. Eq. (9) calculates the average worst-case capital loss over a specific period, while Eq. (10) requires that the average worst-case capital loss be positive. Eq. (11) introduces the budget constraint, ensuring that the sum of the total investment ratios equals 1. Finally, constraint (12) specifies that short selling is not allowed, preventing the investment ratios for each asset from assuming negative values.

Constraints

In the development of a practical portfolio model, it is essential to take into account factors beyond merely risk and returns. This consideration allows the mathematical model to better reflect real-world conditions and yield more realistic results.

Therefore, this study includes cardinality constraints and Floor and Ceiling Constraints in the proposed model. The subsequent section elucidates each of these components.

Definition 2. Cardinality Constraint

In this study, the proposed model includes cardinality constraints, which play a crucial role in the construction of an optimal portfolio. The cardinality constraint specifies the maximum number of assets that may be included in the portfolio. In this constraint, a binary variable Z_i represents the selection status of each asset. The expression for the cardinality constraint is as follows:

$$\sum_{i=1}^{N} Z_i = K \tag{13}$$

$$Z_i \in \{0.1\} \qquad i = 1.2. \dots . n \tag{14}$$

Definition 3. Floor and Ceiling Constraints

The limits on the minimum and maximum investment amounts for each asset within a portfolio can be captured using ceiling and floor constraints. These constraints establish the upper and lower limits for allocating funds to each asset. One possible representation of the ceiling and floor constraints is as follows:

$$l_i Z_i \le x_i \le u_i Z_i . \quad i = 1.2. \dots . n \tag{15}$$

$$0 \le l_i \le u_i \le 1 \tag{16}$$

Definition 4. The proposed CDaR with Practical Constraints

The proposed portfolio optimization model with cardinality and floor and ceiling constraints can be formulated as follows:

$$\min \eta + \frac{1}{(1-\alpha)} \frac{1}{J} \sum_{j=1}^{J} (y_j) \tag{17}$$

subject to

$$\sum_{i=1}^{n} \widetilde{\mu_i} x_i = \widetilde{\mu_p} \tag{18}$$

$$y_j \ge \left\{ \sum_{i=1}^{n} \left(\sum_{t=1}^{k} r_{it} \right) x_i \right\} - \left\{ \sum_{i=1}^{n} \left(\sum_{t=1}^{j} r_{it} \right) x_i \right\} - \eta \tag{19}$$

$$y_j \ge 0 \tag{20}$$

$$\sum_{i=1}^{N} Z_i = K \tag{21}$$

$$l_i Z_i \le x_i \le u_i Z_i . \quad i = 1.2. \dots . n \tag{22}$$

$$Z_i \in \{0.1\} \qquad i = 1.2. \dots . n \tag{23}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{24}$$

$$x_j \ge 0. \quad i = 1.2. \dots . n \tag{25}$$

Definition 5. Fuzzy Programming

Fuzzy programming has been applied in multiple domains, including production planning, energy investment, water management, and financial engineering. In the framework of single-objective problems, two prevalent methodologies are employed, one of which is the approach developed by Jimenez. This method seeks to maximize the expected value of functions while upholding a predetermined level of uncertainty for fuzzy constraints. When endeavoring to maximize these functions, a representation of multi-objective fuzzy programming can be expressed through Eq. (26) and Eq. (27).

$$\max \sum_{J=1}^{n} R_{ij} x_j \quad \forall_i = 1.2. \ldots . m \tag{26}$$

subject to

$$\sum_{i=1}^{n} r_i x_i = \tilde{b}_k \quad \forall_i = 1.2. \ldots . k \tag{27}$$

$$x \in X$$

Proposed Solution Approach

Following the formulation of the primary model of the problem, certain parameters within the model demonstrate fuzzy characteristics, including the values associated with the right-hand side and the technological coefficients that constrain the problem. A two-stage methodology is employed to address the model presented in this paper. In the first stage, the original fuzzy model is converted into a corresponding auxiliary deterministic model. Subsequently, a fuzzy approach is utilized in the second stage to obtain the desired final solution. The specific method outlined in this article is referred to as the Jimenez method, which will be elaborated upon in the following section.

Definition 6. The Jimenez method

In the context of this study, the methodology utilized by Jimenez et al. was adopted to convert the problem involving imprecise coefficients into an equivalent deterministic model. This approach provides significant computational efficiency by maintaining linearity without the need for additional objective functions or inequality constraints. To represent the imprecise nature of the fuzzy parameters in the problem, a triangular fuzzy distribution was chosen for its computational efficiency and ease of data collection. Assuming a triangular fuzzy number, its membership function, denoted as $\mu_{\grave{c}}(x)$, can be expressed using relation (28).

$$\mu_{\grave{c}}(x) = \begin{cases} f_c(x) = \dfrac{x - c^p}{c^m - c^p} & \text{if } c^p \le x \le c^m \\ 1 & \text{if } x = c^m \\ g_c(x) = \dfrac{c^o - x}{c^o - c^m} & \text{if } c^m \le x \le c^o \end{cases} \tag{28}$$

Furthermore, the anticipated interval, denoted as $EI(\grave{c})$, and the anticipated value, denoted as $EV(\grave{c})$, of the fuzzy number is defined using Eq. (29) and Eq. (30), respectively.

$$EI(\grave{c}) = [E_1^c . E_2^c] = \left[\int_0^1 f_c^{-1}(x)dx . \int_0^1 g_c^{-1}(x)dx\right], \tag{29}$$

$$EV(\grave{c}) = \frac{E_1^c + E_2^c}{2}. \tag{30}$$

Given the adoption of the triangular fuzzy distribution for representing the parameters, we will have Eqs. (31-32):

$$EI(\tilde{c}) = \left[\frac{1}{2}(c^p + c^m) . \frac{1}{2}(c^m + c^c)\right] \tag{31}$$

$$EV(\tilde{c}) = \frac{c^p + 2c^m + c^o}{4} \tag{32}$$

Now let us contemplate the fuzzy mathematical programming model, which is formulated based on Eq. (33) and encompasses the inclusion of fuzzy parameters:

$$\begin{aligned} \min \quad & & Z = \tilde{c}x \\ \text{s.t} \quad & \tilde{a}_i x \ge \grave{b}_i & i = 1. \ldots . l \\ & \tilde{a}_i x = \tilde{b}_i & i = 1 + 1. \ldots . m \\ & & x \ge 0 \end{aligned} \tag{33}$$

The imprecision and uncertainty associated with the problem's parameters necessitate the comparison of fuzzy numbers, giving rise to two significant concerns: feasibility and optimality. Consequently, it becomes imperative to address the following two questions, as outlined by Jimenez et al.:

I. How should the decision vector xx be characterized when the constraints include fuzzy numbers?

II.How should the optimality of the objective function be determined in the presence of fuzzy coefficients?

By the Jimenez ranking method, Eq. (34) is formulated for each pair of fuzzy numbers:

$$\mu_M(\grave{a}.\grave{b}) = \begin{cases} 0 & \text{if } E_2^a - E_1^a < 0 \\ \dfrac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & \text{if } 0 \in [E_1^a - E_2^b . E_2^a - E_1^b \\ 1 & \text{if } E_1^a - E_2^b > 0 \end{cases} \quad (34)$$

According to the approach proposed by Jimenez et al., when $\alpha \le \mu_M(\grave{a}_i.\grave{b}_i)$, it indicates that à is at least as great as $\grave{b}$ in degree α. This relationship is denoted as $\grave{a} \ge a\grave{b}$. In the context of their method, the decision vector $x \in \mathbb{R}$ is considered feasible in degree α if condition (35) is satisfied.

$$\min_{i=1.....m}\{\mu_M(\grave{a}_i x.\grave{b}_i)\} = \alpha \quad (35)$$

Thus, about the constraints presented in problem (33), we can establish Eq. (36) as follows:

$$\frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i}} \ge \alpha \qquad i = 1. \dots . l \quad (36)$$

After performing simplifications, relation (36) can be transformed into relation (37) as follows:

$$[(1-\alpha)\cdot E_2^{a_i} + \alpha \cdot E_1^{a_i}]x \ge \alpha \cdot E_2^{b_i} + (1-\alpha)\cdot E_1^{b_i}. \quad i = 1. \dots . m \quad (37)$$

The Eq. (38) will also be valid in the case of a tie condition.

$$\grave{a} \ge_{\alpha/2} \grave{b} \,.\, \grave{a} \le_{\alpha/2} \grave{b} \quad (38)$$

The equation (38) can be reformulated as relation (39):

$$\frac{\alpha}{2} \le \mu_M(\grave{a}.\grave{b}) \le 1 - \frac{\alpha}{2} \quad (39)$$

The solution x^0 is considered a satisfactory optimal solution for the model if relation (40) holds.

$$\mu_M(\grave{c}x.\grave{c}x^0) \ge \frac{1}{2} \quad (40)$$

Hence, the solution x^0 offers an improved solution compared to other feasible vectors (in terms of minimization) by at least a $1/2$ degree. Furthermore, we can establish a relation (41) as follows:

$$\grave{c}x \ge_{1/2} \grave{c}x^0 \quad (41)$$

By utilizing the aforementioned relations, we can deduce the following:

$$\frac{E_2^{cx} - E_1^{cx^0}}{E_2^{cx} - E_1^{cx} + E_2^{cx^0} - E_1^{cx^0}} \ge \frac{1}{2} \quad (42)$$

Or

$$\frac{E_2^{cx} + E_1^{cx}}{2} \ge \frac{E_2^{cx^0} + E_1^{cx^0}}{2} \quad (43)$$

By incorporating relations (37), (39), and (43) into the model (33), we can derive its parametric $-\alpha$ model, represented as the below problem:

$$\min_{Min} Z = EV(\tilde{c})x \quad (44)$$

$$\text{subject to}: \Big[[(1-\alpha)\cdot E_2^{a_i} + \alpha \cdot E_1^{a_i}]x \ge \alpha \cdot E_2^{b_i} + (1-\alpha)\cdot E_1^{b_i}. \quad i = 1. \dots . m \quad (45)$$

$$\left[\left(1-\frac{\alpha}{2}\right)\cdot E_2^{a_i}+\frac{\alpha}{2}\cdot E_1^{a_i}\right]x \geq \frac{\alpha}{2}E_2^{b_i}+\left(1-\frac{\alpha}{2}\right)\cdot E_1^{b_i}\ .\ i=l+1.\dots.m \quad (46)$$

$$Minz_1=\left(C_\beta^m-C_\beta^p\right)X \quad (47)$$

$$\dots\left[\left[\frac{\alpha}{2}\cdot E_2^{a_i}+\left(1-\frac{\alpha}{2}\right)\cdot E_1^{a_i}\right]x \leq \left(1-\frac{\alpha}{2}\right)\cdot E_2^{b_i}+\frac{\alpha}{2}\cdot E_1^{b_i}\ .\ i=l+1.\dots.m\right. \quad (48)$$

$$x \geq 0 \quad (49)$$

The parameter β signifies the minimum level that the decision-maker considers acceptable for establishing feasible constraints. As described in the earlier section, we can create an additional deterministic model that is equal to the primary problem model, denoted as equation (38). The additional problem will contain a larger set of constraints compared to the main problem, due to the conversion of each equality constraint in the primary model into two inequality constraints in the equivalent additional model.

Fuzzy solution approach

Zimmerman introduced the fuzzy solution technique for multi-objective linear programming problems. This method serves as the basis for all subsequent approaches in multi-objective linear programming. Zimmerman (1978) and later Lai and Huang (1994) established the groundwork by creating the balance table and defining fuzzy membership functions for the objective functions. In this paper, Zimmerman's method is utilized to solve the deterministic model presented. The procedure of this method involves the following steps:

- The first stage entails identifying the ideal and anti-ideal solutions for the objective functions, which is achieved through the resolution of three separate single-objective models.

$$Minz_1=\left(C_\beta^m-C_\beta^p\right)X \quad (52)$$

$$Maxz_2=C_\beta^m X \quad (53)$$

$$Maxz_3=\left(C_\beta^O-C_\beta^m\right)X \quad (54)$$

Table 1 presents the financial status of various variables used in the analysis of multi-objective linear programming. It serves as a crucial component of the fuzzy solution approach, which is designed to optimize multiple objectives in a deterministic model.

Table 1
Statement of Financial Position

x_i^*		z_1	z_2	z_3
X_1^*	$Minz_1$	Z_1^{PIS}	$Z_2(X_1^*)$	$Z_3(X_1^*)$
X_2^*	$Maxz_2$	$Z_1(X_2^*)$	Z_2^{PIS}	$Z_3(X_2^*)$
X_3^*	$Maxz_3$	$Z_1(X_3^*)$	$Z_2(X_3^*)$	Z_3^{PIS}
Zi^{NIS}		$Max\{Z_1(X_2^*).Z_1(X_3^*)\}$	$Min\{Z_2(X_1^*).Z_2(X^*{}_3)\}$	$Min\{Z_3(X_1^*).Z_3(X^*{}_2)\}$

- Structure of the Table:
- Variables (x_i^*): Each row corresponds to a specific decision variable (e.g., X_1^*, X_2^*, X_3^*), representing different assets or liabilities in the financial context.
- Objective Functions (Z_1, Z_2, Z_3): The columns represent different objective functions that the model aims to optimize. These functions are as follows:
  - Z_1: Typically represents a minimization objective.
  - Z_2 and Z_3: Represent maximization objectives.

Row Breakdown:
- For each variable variable (e.g., X_1^*, X_2^*, X_3^*), the table displays:
  - The value associated with the minimization of z1z1 or the maximization of Z_2 and Z_3
  - Corresponding membership values (e.g., Z_1^{PIS} , Z_2, and Z_3) provide insights into the suitability of each solution based on fuzzy set theory.
- The final row, labeled Zi^{NIS} , summarizes the overall evaluation by applying maximum and minimum functions across the values from the previous rows. This includes:
  - The maximum value from the membership functions of Z_1 for certain variables.
  - The minimum values from the membership functions of Z_2 and Z_3 indicate the lowest performance scenarios for those objectives.

Table 1 not only aids in visualizing the relationships between different decision variables and their corresponding objectives but also lays the groundwork for applying fuzzy logic in decision-making. By analyzing these values, researchers and practitioners can derive optimal solutions that balance competing financial objectives effectively.

- The second step in the fuzzy solution approach entails the essential task of defining the membership function for each objective function. This process is vital for measuring the extent to which each possible solution meets the established objectives, based on the variations presented in the balance table. The membership functions facilitate the integration of fuzzy logic.

These functions are crucial as they convert the objective values into a membership scale, which indicates how closely each solution aligns with the desired outcomes. By quantifying the objectives in this way, decision-makers can effectively apply principles of fuzzy logic to assess and compare potential solutions, ultimately leading to more informed and balanced financial decision-making. The membership functions for the objective functions Z_1, Z_2, and Z_3 are defined as follows:

$$\boldsymbol{\mu_{z1}(x)=\begin{cases}1 & Z_1(X)\le Z_{1\beta}{}^{PIS}\\ \dfrac{Z_{1\beta}{}^{NIS}-\left(C_\beta^m-C_\beta^P\right)X}{Z_1{}^{NIS}-Z_1{}^{PIS}} & Z_{1\beta}{}^{PIS}\le Z_1(X)\le Z_{1\beta}^{NIS}\\ 0 & Z_1(X)\ge Z_{1\beta}{}^{NIS}\end{cases}} \quad (55)$$

$$\mu_{Z2}(x)=\begin{cases}1 & Z_2(X)\ge Z_{2\beta}{}^{PIS}\\ \dfrac{C^m{}_\beta X-Z_{2\beta}{}^{NIS}}{Z_2{}^{PIS}-Z_2{}^{NIS}} & Z_{2\beta}{}^{NIS}\le Z_2(X)\le Z_{2\beta}{}^{PIS}\\ 0 & Z_2(X)\le Z_{2\beta}{}^{NIS}\end{cases} \quad (56)$$

$$\mu_{Z3}(x)=\begin{cases}1 & Z_3(X)\ge Z_{3\beta}^{PIS}\\ \dfrac{\left(C_\beta^O-C_\beta^m\right)X-Z_{3\beta}{}^{NIS}}{Z_3{}^{PIS}-Z_3{}^{NIS}} & Z_{3\beta}^{NIS}\le Z_3(X)\le Z_{3\beta}{}^{PIS}\\ 0 & Z_3(X)\le Z_{3\beta}{}^{NIS}\end{cases} \quad (57)$$

- The third step: entails transforming the original multi-objective model into an equivalent single-objective model using an integration function. The integration function facilitates this conversion process. Additionally, the minimum satisfaction level for the objective function is denoted as λ.

$$\boldsymbol{\lambda=\min\{\mu_{zi}\,(x)\}} \quad (58)$$

$$\textbf{max}\ \boldsymbol{\lambda}\ \textbf{Subject to} \quad (59)$$

$$\boldsymbol{\left(C_\beta^m-C_\beta^P\right)X+\lambda\left(Z_{1\beta}{}^{NIS}-Z_{1\beta}{}^{PIS}\right)\le Z_{1\beta}{}^{NIS}} \quad (60)$$

$$\boldsymbol{C_{\beta\,\beta}X-\lambda\left(Z_{2\beta}{}^{NIS}-Z_{2\beta}{}^{PIS}\right)\ge Z_{2\beta}{}^{NIS}} \quad (61)$$

$$\boldsymbol{\left(C_\beta^O-C^m{}_\beta\right)X-\lambda\left(Z_{3\beta}{}^{NIS}-Z_{3\beta}{}^{PIS}\right)\ge Z_{3\beta}{}^{NIS}} \quad (62)$$

$$\boldsymbol{x\in F_X} \quad (63)$$

$$\boldsymbol{\lambda\in[0.1]} \quad (64)$$

It is crucial to incorporate Eq. (49) as a deterministic constraint into the original problem to ensure precision. Consequently, the mathematical formulation of the problem is structured as follows:

$$Z_1=\max\sum_{j=1}^{6} r_i x_j \quad (65)$$

$$Z_2=\min\eta+\frac{1}{(1-\alpha)}\frac{1}{J}\sum_{j=1}^{J}\left(y_j\right) \quad (66)$$

subject to:

$$y_j \geq \left\{\sum_{i=1}^{n}\left(\sum_{t=1}^{k} r_{it}\right)x_i\right\} - \left\{\sum_{i=1}^{n}\left(\sum_{t=1}^{j} r_{it}\right)x_i\right\} - \eta \tag{67}$$

$$\sum_{i=1}^{n} r_i {x_i}^2 - \sum_{t=1}^{k} b_i\ x_i \geq d \tag{68}$$

$$y_j \geq 0 \tag{69}$$

$$\sum_{i=1}^{N} Z_i = K \tag{70}$$

$$l_i Z_i \leq x_i \leq u_i Z_i. \quad i = 1.2.\dots.n \tag{71}$$

$$Z_i \in \{0.1\} \qquad i = 1.2.\dots.n \tag{72}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{73}$$

$$x_j \geq 0. \quad i = 1.2.\dots n \tag{74}$$

4. Data collection and analysis

To evaluate the various models, our research utilized data acquired from the Tehran Stock Exchange (TSE). The assets chosen for examination are detailed in Table 2. A comprehensive and systematic screening process was implemented to select these assets based on specific criteria.

The initial criterion required a minimum market capitalization of 100 billion units for the stocks under consideration. In addition, the stocks had to possess a trading history of at least 9 months within the year to ensure reliability and relevance. Moreover, an intentional effort was made to ensure the inclusion of stocks from diverse industries, providing a more representative sample for analysis.

This data collection approach represents a significant innovation in our study, as it guarantees that the dataset contains stocks that meet rigorous criteria while also improving the comprehensiveness and relevance of the analysis. By utilizing these clearly defined screening criteria, we have established a strong dataset that enables a detailed and comparative analysis of portfolio optimization models with real-world data from the TSE.

Table 2
Selected asset data from TSE

Asset		
SEFH	BKSZ	PKLJ
BAMA	BENC	FOLD
IKHR	FRBZ	IKCO
BMLT	KALA	
BIEJ	OFRS	
HFRS	SAHD	
KSHI	HWEB	
MKBT	MAPN	

5. Results Analysis

This section provides a comprehensive analysis of the results obtained from employing an optimally selected portfolio sample, consisting of shares from particular companies listed on the Tehran Stock Exchange (TSE). The analysis incorporates random parameters to account for the inherent uncertainties present in financial markets. To ensure the robustness of our findings, the model is developed and assessed under two distinct scenarios: conditions of certainty and uncertainty.

To identify the optimal portfolio, the proposed model is systematically solved using various values of the parameter β. As presented in Table 3, this table displays the portfolio values associated with each β value, along with the optimal stocks selected and their corresponding allocations. In this context, β is defined as the minimum acceptable threshold, which represents potential constraints established by the decision maker to signify their desired level of confidence regarding the investment.

To further refine the modeling process, a cardinality constraint, denoted as $k = 3$, has been imposed. This constraint limits the optimal portfolio to a maximum of three assets. Additionally, the research sets specific bounds for the investment allocations, with a lower limit of $l_i = 0.1$ and an upper limit of $u_i = 0.45$ for each asset *i*. These parameters are essential for establishing the investment constraints and provide a structured framework for effective portfolio optimization.

Table 3

Results of Model Solutions for Various β Values

β value	Asset	value	Portfolio Value	β value	Asset	value	Portfolio Value
	HFRS	0.45			HFRS	0.45	
0.6	KALA	0.1	0.32	0.8	KALA	0.1	**0.23**
	OFRS	0.45			OFRS	0.45	
	HFRS	0.45			HFRS	0.45	
0.7	KALA	0.1	0.27	0.9	KALA	0.1	**0.18**
	OFRS	0.45			OFRS	0.45	

After analyzing the results obtained from solving the model presented in Table 3, it is apparent that the portfolio displays varying values across different β values. Specifically, as the minimum acceptable threshold for potential constraints, represented by β, decreases, the value of the constructed portfolio shows an increase. Upon evaluating the optimal stocks within each portfolio, it is observed that the value of β does not influence stock selection, as evidenced by the consistent inclusion of options such as HFRS, OFRS, and KALA in all portfolios.

To validate the study, the model is developed and evaluated under two distinct conditions: uncertainty and certainty. Tables 4 and 5 present and explain the results for both scenarios, confirming the research findings.

In this section, we seek to validate the findings presented in the previous sections of the research by conducting problem modeling under conditions of certainty. This approach allows for a more straightforward assessment of the optimal portfolio configuration, facilitating comparisons with results obtained under conditions of uncertainty.

Table 4 illustrates that the optimal stocks selected for the portfolio remain consistent between the certainty and uncertainty scenarios, specifically highlighting the inclusion of KALA, HFRS, and OFRS shares. Notably, the allocation percentages of these stocks exhibit remarkable stability across both analyzed portfolios, indicating a strong resistance to variability in market conditions.

The consistency in stock selection not only strengthens the robustness of the proposed model but also highlights the reliability of these assets within the investment framework. This stability further indicates that the selected assets have intrinsic characteristics that make them advantageous regardless of the level of market certainty, which is a crucial factor for investors seeking to construct resilient portfolios.

Table 4

Preferred Stocks and their Quantities in the Portfolio under Certainty and Fuzzy Uncertainty Scenarios

Uncertainty		Certainty	
Asset	value	Asset	value
HFRS	0.45	HFRS	**0.45**
KALA	0.1	KALA	**0.1**
OFRS	0.45	OFRS	**0.45**

Table 5 delineates the portfolio's valuation under conditions of certainty and fuzzy uncertainty. The results obtained from solving the model under both certainty and fuzzy uncertainty conditions, as outlined in Table 5, reveal a higher portfolio valuation under certainty, also for fuzzy uncertainty portfolios with smaller values of β, signifying a more favorable outcome. As indicated in Table 4, the analysis emphasizes the significance of KALA, HFRS, and OFRS shares as fundamental components of the optimal portfolio. The assets allocated to this portfolio maintain the specified ratios, with the maximum value of the examined stocks in the portfolio reaching 0.32.

Table 5

Portfolio Valuation under Certainty and Fuzzy Uncertainty Conditions

β value	Portfolio value under Fuzzy Uncertainty	Portfolio value under Certainty
0.6	0.32	
0.7	0.27	0.62
0.8	0.23	
0.9	0.18	

In conclusion, the findings from this section corroborate the initial research results, demonstrating that the optimal portfolio construction is not only effective under certainty but also maintains its integrity when subjected to varying levels of uncertainty.

Table 4 and Table 5 provide a comprehensive overview of the allocation of each share within the portfolio, in conjunction with the corresponding optimal portfolio values for both certainty and uncertainty scenarios. These tables serve as pivotal tools for assessing the robustness of the selected optimal shares and their allocations.

Visualizing Optimal Allocation Proportions

Fig. 1 illustrates the optimal allocation proportions for the portfolio, highlighting the exclusion of stocks that do not qualify as optimal choices. According to Table 4, the stocks KALA, HFRS, and OFRS form the optimal portfolio, with allocation values of 0.10, 0.45, and 0.45, respectively. The visual representation in Figure 1 emphasizes the relative investment levels assigned to each stock, reflecting a strategic approach to maximizing returns while managing risk in uncertain market conditions. This framework demonstrates how selective asset allocation can enhance the portfolio's overall performance.

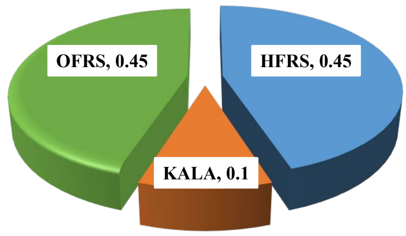

Fig. 1. Allocation Ratio of Shares in the Optimal Portfolio Amidst Uncertainty

Fig. 2 serves as a visual representation of these allocation proportions, effectively illustrating the distribution of investments among the selected assets within the portfolio under conditions characterized by certainty. The visual depiction not only enhances the clarity of the allocation strategy but also underscores the relative significance of each stock within the optimal portfolio framework. By clearly delineating the allocation ratios, Fig. 2 allows for a straightforward understanding of how these stocks are prioritized, facilitating an assessment of their roles in achieving optimal investment outcomes.

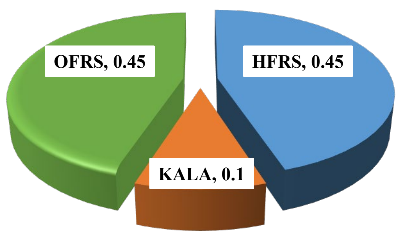

Fig. 2. Allocation Ratio of Shares in the Optimal Portfolio Amidst Uncertainty

This graphical representation complements the quantitative data provided in Table 4, reinforcing the idea that the chosen allocation strategy is not solely based on numerical values, but also involves careful consideration of each asset's potential to contribute to the portfolio's success.

A comparison between the results of the deterministic and non-deterministic models, as shown in Tables 4 and 5, indicates that amid uncertainty, shares KALA, HFRS, and OFRS are incorporated into the portfolio with weights of 0.10, 0.45, and 0.45, respectively. Ultimately, the optimal value of the portfolio amounts to 0.32. Conversely, in the deterministic scenario depicted in Tables 5 and 6, the chosen stocks for the portfolio remain the same, namely KALA, HFRS, and OFRS, with identical weights of 0.10, 0.45, and 0.45. The optimal value of the portfolio in this case totals 0.62, surpassing the value obtained from the uncertain model. This difference highlights the increased value of the portfolio under conditions of certainty.

6. Conclusion and discussion

The forthcoming study aims to explore the optimization of portfolios consisting of shares from selected companies listed on the Tehran Stock Exchange, particularly focusing on sectors currently experiencing growth. This research is situated within a context marked by significant stock market volatility, a phenomenon that has become increasingly prominent in recent years. It also recognizes the inherent risks associated with investing in such a volatile environment. To address these challenges effectively, the study employs the Conditional Value-at-Risk (CDaR) measure as a prudent risk metric for evaluating investments within the stock market. This risk measure is classified as a downside risk metric, making it suitable for both risk assessment and portfolio optimization, as it considers the probability distribution of asset returns.

To enhance the applicability and relevance of the research, the study integrates additional elements reflecting investor preferences, including constraints such as cardinality restrictions—limits on the number of assets included in the portfolio—as well as upper and lower bounds that operate under conditions of uncertainty. The incorporation of fuzzy programming techniques is a fundamental aspect of this study, enabling the assessment of portfolios in uncertain contexts and facilitating a more nuanced analysis of potential outcomes. The planning approach proposed by Jimenez is utilized to effectively implement these constraints, while problem-solving is supported through Zimmerman's method.

The findings from this comprehensive analysis underscore the effectiveness of the proposed model in constructing an optimal portfolio. Notably, the results reveal that the portfolio's value is significantly higher under conditions of certainty compared to those characterized by uncertainty. This conclusion remains valid even when both portfolios are developed under similar circumstances, differing only in their respective levels of certainty. Such findings suggest that portfolios inherently possess greater intrinsic value when situated in certain environments, indicating that certainty plays a crucial role in enhancing asset valuation.

The outcomes of this research are expected to provide valuable insights for various stakeholders in the finance sector, including scholars, individual investors, investment fund managers, and other market participants. Looking forward, future research initiatives may focus on examining and comparing diverse alternative portfolio optimization models applicable to the stock market. These studies could employ various methodologies to effectively address uncertainties in mathematical modeling. Additionally, to improve the alignment of mathematical models and their results with the complexities of real-world market dynamics, future investigations might consider incorporating practical constraints, such as liquidity limitations and transaction costs, into the portfolio optimization framework. Such enhancements would significantly improve the model's relevance, ensuring its applicability and effectiveness in real-world investment scenarios.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

In the course of preparing this work, the authors utilized OpenAI's Chat GPT tool to edit and compose certain sections of the paper. Following the use of this service, the authors conducted a thorough review and made necessary revisions to the content, assuming full responsibility for the publication's content.

References

Ammar, E., & Khalifa, H. A. (2003). Fuzzy portfolio optimization is a quadratic programming approach. *Chaos, Solitons & Fractals*, *18*(5), 1045-1054 .

Ammar, E. E. (2008). On solutions of fuzzy random multiobjective quadratic programming with applications in a portfolio problem. *Information Sciences*, *178*(2), 468-484 .

Björk, K.-M. (2009). An analytical solution to a fuzzy economic order quantity problem. *International journal of approximate reasoning*, *50*(3), 485-493 .

Chekhlov, A., Uryasev, S., & Zabarankin, M .(2004) .Portfolio optimization with drawdown constraints. In *Supply chain and finance* (pp. 209-228). World Scientific .

Chen, L.-H., & Huang, L. (2009). Portfolio optimization of equity mutual funds with fuzzy return rates and risks. *Expert Systems with Applications*, *36*(2), 3720-3727 .

Chen, S.-P. (2004). Parametric nonlinear programming for analyzing fuzzy queues with finite capacity. *European Journal of Operational Research*, *157*(2), 429-438 .

Eskorouchi, A., Mohammadi, E., & Sadjadi, S. J. (2022). Robust Portfolio Optimization based on Evidence Theory. *Scientia Iranica*.

Fei, W. (2007). Optimal consumption and portfolio choice with ambiguity and anticipation. *Information Sciences*, *177*(23), 5178-5190 .

Ghanbari, H., Safari, M., Ghousi, R., Mohammadi, E., & Nakharutai, N. (2023). Bibliometric analysis of risk measures for portfolio optimization. *Accounting*, *9*(2), 95-108 .

Gupta, P., Mehlawat, M. K., & Saxena, A. (2008). Asset portfolio optimization using fuzzy mathematical programming. *Information Sciences*, *178*(6), 1734-1755 .

Huang, S.-H., Miao, Y.-H., & Hsiao, Y.-T. (2021). Novel deep reinforcement algorithm with adaptive sampling strategy for continuous portfolio optimization. *IEEE Access*, *9*, 77371-77385 .

Khosravi, A., Sadjadi, S., & Ghanbari, H. (2024). A bibliometric analysis and visualization of the scientific publications on multi-period portfolio optimization: From the current status to future directions. *Accounting*, *10*(3), 107-120.

Konno, H., & Yamazaki, H. (1991). Mean-absolute deviation portfolio optimization model and its applications to the Tokyo stock market. *Management Science*, *37*(5), 519-531 .

Krokhmal, P., Uryasev, S., & Zrazhevsky, G. (2005). Numerical comparison of conditional value-at-risk and conditional drawdown-at-risk approaches: application to hedge funds. In *Applications of stochastic programming* (pp. 609-631). SIAM .

Li, C., Wu, Y., Lu, Z., Wang, J., & Hu, Y. (2020). A multiperiod multiobjective portfolio selection model with fuzzy random returns for large-scale securities data. *IEEE Transactions on Fuzzy Systems*, *29*(1), 59-74 .

Liu, S.-T. (2005). Fuzzy measures for fuzzy signal-to-noise ratios .

Markowitz, H. (1952). Modern portfolio theory. *Journal of Finance*, *7* .91-77 ,(11)

Markowitz Harry, M. (1959). Portfolio selection: Efficient diversification of investments. *Cowles Foundation Monograph*, *16* .

Perold, A. F. (1984). Large-scale portfolio optimization. *Management Science*, *30*(10), 1143-1160 .

Rockafellar, R. T., & Uryasev, S. (2000). Optimization of conditional value-at-risk. *Journal of Risk*, *2*, 21-42 .

Sakawa, M., Utaka, J., Inniguchi, I., Shiromaru, I., Suginohara, N., & Inoue, T. (1993). Hot parts operating schedule of gas turbines by genetic algorithms and fuzzy satisficing methods. Proceedings of 1993 International Conference on Neural Networks (IJCNN-93-Nagoya, Japan) ,

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, *19*(3), 425-442 .

Sharpe, W. F. (1967). A linear programming algorithm for mutual fund portfolio selection. *Management Science*, *13*(7), 499-510 .

Sharpe, W. F. (1971). A linear programming approximation for the general portfolio analysis problem. *Journal of Financial and Quantitative Analysis*, *6*(5), 1263-1275 .

Stone, B. K. (1973). A linear programming formulation of the general portfolio selection problem. *Journal of Financial and Quantitative Analysis*, *8*(4), 621-636 .

Vercher, E. (2008). Portfolios with fuzzy returns: selection strategies based on semi-infinite programming. *Journal of computational and applied mathematics*, *217*(2), 381-393 .

Vijayan, T., & Kumaran, M. (2009). Fuzzy economic order time models with random demand. *International journal of approximate reasoning*, *50*(3), 529-540 .

Wu, M.-E., Syu, J.-H., Lin, J. C.-W., & Ho, J.-M. (2021). Effective fuzzy system for qualifying the characteristics of stocks by random trading. *IEEE Transactions on Fuzzy Systems*, *30*(8), 3152-3165 .

Zadeh, L. A. (1978). Fuzzy sets as a basis for a theory of possibility. *Fuzzy sets and systems*, *1*(1), 3-28 .

Zarezade, R., Ghousi, G., & Mohammadi, E. (2024). Spillover effects of volatility between the Chinese stock market and selected emerging economies in the Middle East: A conditional correlation analysis with portfolio optimization perspective. *Accounting*, *10*(2), 97-106 .

Zarezade, R., Ghousi, R., Mohammadi, E., & Ghanbari, H. (2024). Chance-constrained programming for Cryptocurrency portfolio optimization using Conditional Drawdown at Risk. *Journal of Industrial and Systems Engineering*, *16*(2), 130-153 .

Zimmerman, J. (1978). Topographic generation of residual circulation by oscillatory (tidal) currents. *Geophysical & Astrophysical Fluid Dynamics*, *11*(1), 35-47 .